Vortex interactions in two component Ginzburg-Landau theory and type 1.5 superconductivity

Martin Speight
University of Leeds
Joint work with Egor Babaev and Johan Carlstrom

February 9, 2011

Two component GL theory

- Two charged scalar fields ψ_1, ψ_2
  - Two component superconductors: condensates of electron Cooper pairs in two different pairing states
  - Liquid metallic hydrogen: electrons and protons

Two component GL theory

- Two charged scalar fields ψ_1, ψ_2
  - Two component superconductors: condensates of electron Cooper pairs in two different pairing states
  - Liquid metallic hydrogen: electrons and protons
- Electromagnetic gauge field $\mathbf{A}$

Two component GL theory

- Two charged scalar fields ψ_1, ψ_2
  - Two component superconductors: condensates of electron Cooper pairs in two different pairing states
  - Liquid metallic hydrogen: electrons and protons
- Electromagnetic gauge field $\mathbf{A}$
- Assume translation invariance in z direction

$$E = \frac{1}{2}\int_{\mathbb{R}^2} |\mathbf{D}\psi_1|^2 + |\mathbf{D}\psi_2|^2 + B^2 + 2V(\psi_1, \psi_2)$$

where $\mathbf{D}\psi = (\nabla - i\mathbf{A})\psi$, $B = \partial_1 A_2 - \partial_2 A_1$

Two component GL theory

- Two charged scalar fields ψ_1, ψ_2
  - Two component superconductors: condensates of electron Cooper pairs in two different pairing states
  - Liquid metallic hydrogen: electrons and protons
- Electromagnetic gauge field $\mathbf{A}$
- Assume translation invariance in z direction

$$E = \frac{1}{2}\int_{\mathbb{R}^2} |\mathbf{D}\psi_1|^2 + |\mathbf{D}\psi_2|^2 + B^2 + 2V(\psi_1, \psi_2)$$

where $\mathbf{D}\psi = (\nabla - i\mathbf{A})\psi$, $B = \partial_1 A_2 - \partial_2 A_1$

- Gauge invariance: E invariant under

$$\psi_a \mapsto e^{i\chi}\psi_a, \qquad \mathbf{A} \mapsto \mathbf{A} + \nabla\chi$$

$\Rightarrow V(|\psi_1|, |\psi_2|, \theta)$ where $\theta = \arg(\psi_1) - \arg(\psi_2)$.

Two component GL theory

- Interesting examples

$$
\begin{aligned}
V &= V_0 + \alpha_1|\psi_1|^2 + \frac{\beta_1}{2}|\psi_1|^4 + \alpha_2|\psi_2|^2 + \frac{\beta_2}{2}|\psi_2|^4 \\
&\quad (\alpha_1, \alpha_2 < 0) \\
V &= \text{above} - \frac{\eta}{2}(\psi_1\overline{\psi_2} - \overline{\psi_1}\psi_2)
\end{aligned}
$$

etc

Two component GL theory

- Interesting examples

$$
\begin{aligned}
V &= V_0 + \alpha_1|\psi_1|^2 + \frac{\beta_1}{2}|\psi_1|^4 + \alpha_2|\psi_2|^2 + \frac{\beta_2}{2}|\psi_2|^4 \\
&\quad (\alpha_1, \alpha_2 < 0) \\
V &= \text{above} - \frac{\eta}{2}(\psi_1\overline{\psi_2} - \overline{\psi_1}\psi_2)
\end{aligned}
$$

  etc

- Want $V : \mathbb{C}^2 \to \mathbb{R}$ to have an *unstable* critical point at $(0,0)$ and a *global minimum* at $\psi_1, \psi_2 \neq 0$. WLOG, can assume global min occurs at $\psi_1 = u_1 > 0$, $\psi_2 = u_2 > 0$ and has $V(u_1, u_2) = 0$.

Two component GL theory

- Interesting examples

$$\begin{aligned} V &= V_0 + \alpha_1|\psi_1|^2 + \frac{\beta_1}{2}|\psi_1|^4 + \alpha_2|\psi_2|^2 + \frac{\beta_2}{2}|\psi_2|^4 \\ &\quad (\alpha_1, \alpha_2 < 0) \\ V &= \text{above} - \frac{\eta}{2}(\psi_1\overline{\psi_2} - \overline{\psi_1}\psi_2) \end{aligned}$$

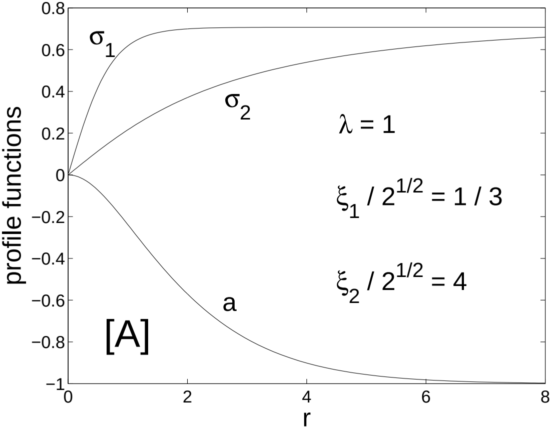
  etc

- Want $V : \mathbb{C}^2 \to \mathbb{R}$ to have an *unstable* critical point at $(0, 0)$ and a *global minimum* at $\psi_1, \psi_2 \neq 0$. WLOG, can assume global min occurs at $\psi_1 = u_1 > 0$, $\psi_2 = u_2 > 0$ and has $V(u_1, u_2) = 0$.
- Model supports **vortex solutions**

$$\psi_a = \sigma_a(r)e^{i\theta}, \qquad \mathbf{A} = \frac{a(r)}{r}(-\sin\theta, \cos\theta)$$

  with real profile functions σ_1, σ_2, a interpolating between 0 (at $r = 0$) and $u_1, u_2, 1$ respectively (as $r \to \infty$).

Two component GL theory



Two component GL theory

$$E = \frac{1}{2}\int_{\mathbb{R}^2} |\mathbf{D}\psi_1|^2 + |\mathbf{D}\psi_2|^2 + B^2 + 2V(\psi_1, \psi_2)$$

- Flux quantization: $\psi_a(r,\theta) \sim u_a e^{i\chi(\theta)}$, $\mathbf{A} \sim \nabla\chi$ as $r \to \infty$
  Stokes's Theorem $\Rightarrow$

$$\Phi = \int_{\mathbb{R}^2} B = \int_{S^1_\infty} \mathbf{A} = 2\pi n$$

where $n =$ winding number of ψ_1 (and ψ_2).

Two component GL theory

$$E = \frac{1}{2}\int_{\mathbb{R}^2} |\mathbf{D}\psi_1|^2 + |\mathbf{D}\psi_2|^2 + B^2 + 2V(\psi_1, \psi_2)$$

- Flux quantization: $\psi_a(r,\theta) \sim u_a e^{i\chi(\theta)}$, $\mathbf{A} \sim \nabla\chi$ as $r \to \infty$
  Stokes's Theorem $\Rightarrow$

$$\Phi = \int_{\mathbb{R}^2} B = \int_{S^1_\infty} \mathbf{A} = 2\pi n$$

  where $n =$ winding number of ψ_1 (and ψ_2).
- Vortex has $n = 1$, hence $\Phi = 2\pi$. Exponentially spatially localized (flux tube).

Two component GL theory

$$E = \frac{1}{2}\int_{\mathbb{R}^2} |\mathbf{D}\psi_1|^2 + |\mathbf{D}\psi_2|^2 + B^2 + 2V(\psi_1,\psi_2)$$

- Flux quantization: $\psi_a(r,\theta) \sim u_a e^{i\chi(\theta)}$, $\mathbf{A} \sim \nabla\chi$ as $r \to \infty$
  Stokes's Theorem $\Rightarrow$

$$\Phi = \int_{\mathbb{R}^2} B = \int_{S^1_\infty} \mathbf{A} = 2\pi n$$

  where $n =$ winding number of ψ_1 (and ψ_2).
- Vortex has $n = 1$, hence $\Phi = 2\pi$. Exponentially spatially localized (flux tube).
- Intervortex forces?

The abelian Higgs model

- **Single** component GL theory

$$E = \int_{\mathbb{R}^2} \frac{1}{2}|\mathbf{D}\psi|^2 + \frac{1}{2}B^2 - \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4$$

The abelian Higgs model

- **Single** component GL theory

$$E = \int_{\mathbb{R}^2} \frac{1}{2}|\mathbf{D}\psi|^2 + \frac{1}{2}B^2 - \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4$$

- Static case of abelian Higgs model in $\mathbb{R}^{(2,1)}$

$$S = \int_{\mathbb{R}^{(2,1)}} \frac{1}{2}\overline{D_\mu\psi}D^\mu\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\mu^2}{8}(1-|\psi|^2)^2$$

Relativistic field theory in $2+1$ dimensions, $D_\mu = \partial_\mu + iA_\mu$, $\mu = 0,1,2$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ (so $B = F_{12}$).

The abelian Higgs model

- **Single** component GL theory

$$E = \int_{\mathbb{R}^2} \frac{1}{2}|\mathbf{D}\psi|^2 + \frac{1}{2}B^2 - \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4$$

- Static case of abelian Higgs model in $\mathbb{R}^{(2,1)}$

$$S = \int_{\mathbb{R}^{(2,1)}} \frac{1}{2}\overline{D_\mu\psi}D^\mu\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\mu^2}{8}(1-|\psi|^2)^2$$

Relativistic field theory in $2+1$ dimensions, $D_\mu = \partial_\mu + iA_\mu$, $\mu = 0,1,2$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ (so $B = F_{12}$).

- Still has static vortices

$$\psi = \sigma(r)e^{i\theta}, \qquad (A_0, A_1, A_2) = \frac{a(r)}{r}(0, \sin\theta, -\cos\theta)$$

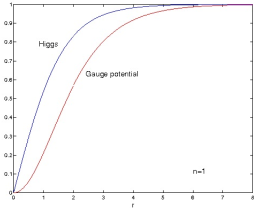
The abelian Higgs model

- Topological solitons: smooth, spatially localized, finite energy solutions of nonlinear relativistic field theory with particle-like behaviour

The abelian Higgs model

- Topological solitons: smooth, spatially localized, finite energy solutions of nonlinear relativistic field theory with particle-like behaviour
  - Can Lorentz boost them
  - Have anti-vortices (winding $n = -1$)
  - Far from the vortex core the fields **look like those induced in a linear theory by a point source at the vortex centre**

Vortex asymptotics



- Asymptotics: for $\mu \leq 2$,

$$\begin{aligned}\sigma(r) &\sim 1 + \frac{q}{2\pi} K_0(\mu r)\\ a(r) &\sim 1 - \frac{m}{2\pi} r K_0'(r)\end{aligned}$$

  where $K_0 =$ modified Bessel function of the second kind, q, m are unknown constants.

- Note $K_0(r) \sim \sqrt{\frac{\pi}{2r}} e^{-r}$

Linearized model

- Idea: replicate far-field of vortex in linearized theory by introducing appropriate point sources.

Linearized model

- Idea: replicate far-field of vortex in linearized theory by introducing appropriate point sources.
- Which linear theory? Linearize AHM about the vacuum: choose real gauge, $\psi = 1 + \varphi$,

$$\mathcal{L} = \frac{1}{2}\partial_\mu\psi\partial^\mu\varphi - \frac{\mu^2}{2}\varphi^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}A_\mu A^\mu + \kappa\psi - j_\mu A^\mu$$

Klein-Gordon-Proca theory: φ scalar boson (Higgs) of mass μ, A^μ vector boson (photon) of mass 1.

Linearized model

- Idea: replicate far-field of vortex in linearized theory by introducing appropriate point sources.
- Which linear theory? Linearize AHM about the vacuum: choose real gauge, $\psi = 1 + \varphi$,

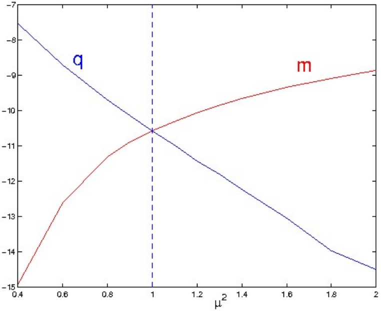
$$\mathcal{L} = \frac{1}{2}\partial_\mu\psi\partial^\mu\varphi - \frac{\mu^2}{2}\varphi^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}A_\mu A^\mu + \kappa\psi - j_\mu A^\mu$$

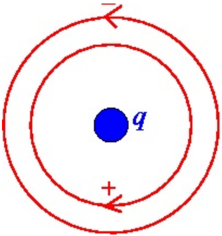
  Klein-Gordon-Proca theory: φ scalar boson (Higgs) of mass μ, A^μ vector boson (photon) of mass 1.
- Asymptotic vortex fields induced by

$$\begin{aligned} \kappa &= q\delta(\mathbf{x}) && \text{scalar monopole, charge } q \\ \mathbf{j} &= -m\mathbf{k} \times \nabla\delta(\mathbf{x}) && \text{magnetic dipole of moment } m\mathbf{k} \end{aligned}$$

  Composite point source, "point vortex"

Point vortices





- At $\mu = 1$, $q = m$. Not a coincidence!

Point vortex interactions

- Deep principle (or leap of faith): since vortex is asymptotically indistinguishable from a point particle carrying sources for a linear theory, the interactions between vortices should be well-approximated, at long range, by those between the corresponding point particles.

Point vortex interactions

- Deep principle (or leap of faith): since vortex is asymptotically indistinguishable from a point particle carrying sources for a linear theory, the interactions between vortices should be well-approximated, at long range, by those between the corresponding point particles.
- The latter are easy to compute: **linear** field theory

$$L_{\rm int} = \int_{\mathbb{R}^2} \{\kappa_{(1)}\psi_{(2)} - j^{\mu}_{(1)} A^{(2)}_{\mu}\} = L_{\psi} + L_A$$

Point vortex interactions

- Deep principle (or leap of faith): since vortex is asymptotically indistinguishable from a point particle carrying sources for a linear theory, the interactions between vortices should be well-approximated, at long range, by those between the corresponding point particles.
- The latter are easy to compute: **linear** field theory

$$L_{\rm int} = \int_{\mathbb{R}^2} \{\kappa_{(1)}\psi_{(2)} - j^{\mu}_{(1)}A^{(2)}_{\mu}\} = L_{\psi} + L_A$$

- Two point vortices at rest at $\mathbf{y}$, $\mathbf{z}$

$$\begin{aligned} L_{\psi} &= \int q\delta(\mathbf{x}-\mathbf{y})\frac{q}{2\pi}K_0(\mu|\mathbf{x}-\mathbf{z}|)\,d^2\mathbf{x} = \frac{q^2}{2\pi}K_0(\mu|\mathbf{y}-\mathbf{z}|) \\ L_A &= \cdots = -\frac{m^2}{2\pi}K_0(|\mathbf{y}-\mathbf{z}|) \end{aligned}$$

Vortex interaction potential

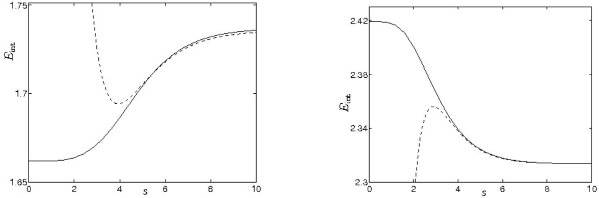
$$V_{\mathrm{int}} = \frac{1}{2\pi}[m^2 K_0(s) - q^2 K_0(\mu s)]$$

- Reproduces familiar trichotomy:
  - $\mu < 1$ attractive - type I
  - $\mu > 1$ repulsive - type II
  - $\mu = 1$ cancel, $V_{\mathrm{int}} = 0$.

Vortex interaction potential

$$V_{\rm int} = \frac{1}{2\pi}[m^2 K_0(s) - q^2 K_0(\mu s)]$$

- Reproduces familiar trichotomy:
  - $\mu < 1$ attractive - type I
  - $\mu > 1$ repulsive - type II
  - $\mu = 1$ cancel, $V_{\rm int} = 0$.
- Cf constrained minimization:



Back to two component model

- Try the same trick
  - Think of TCGL model as static case of TCAHM
  - Think of vortices as topological solitons
  - Replicate vortex asymptotics with point sources in the linearized model
  - Read off asymptotic interaction potential

Back to two component model

- Try the same trick
  - Think of TCGL model as static case of TCAHM
  - Think of vortices as topological solitons
  - Replicate vortex asymptotics with point sources in the linearized model
  - Read off asymptotic interaction potential
- New phenomenon:
  - In one-component case we had two length scales, set by mass of Higgs, μ, and mass of photon, 1
  - In two component case, we have **four**, of which **three** are relevant: two Higgs masses μ_1, μ_2 and the photon mass μ_A
  - Interesting regime: $\mu_1 < \mu_A < \mu_2$
    Can allow **non-monotonic** vortex interaction potential: attractive at long range but repulsive at short range.

Two-component abelian Higgs model

$$\mathcal{L} = \frac{1}{2}\overline{D_\mu\psi_1}D^\mu\psi_1 + \frac{1}{2}\overline{D_\mu\psi_2}D^\mu\psi_2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - V(\psi_1,\psi_2)$$

Two-component abelian Higgs model

$$\mathcal{L} = \frac{1}{2}\overline{D_\mu \psi_1} D^\mu \psi_1 + \frac{1}{2}\overline{D_\mu \psi_2} D^\mu \psi_2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - V(\psi_1, \psi_2)$$

- Linearize about $A = 0$, $\psi_1 = u_1$, $\psi_2 = u_2$ in real ψ_1 gauge.

$$\psi_1 = u_1 + \varphi_1, \qquad \psi_2 = (u_2 + \varphi_2) e^{i\varphi_3}$$

Two-component abelian Higgs model

$$\mathcal{L} = \frac{1}{2}\overline{D_\mu\psi_1}D^\mu\psi_1 + \frac{1}{2}\overline{D_\mu\psi_2}D^\mu\psi_2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - V(\psi_1, \psi_2)$$

- Linearize about $A = 0$, $\psi_1 = u_1$, $\psi_2 = u_2$ in real ψ_1 gauge.

$$\psi_1 = u_1 + \varphi_1, \qquad \psi_2 = (u_2 + \varphi_2)e^{i\varphi_3}$$

- Vortex has $\varphi_3 \equiv 0$, so can drop it

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi_a\partial^\mu\varphi_a - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(u_1^2 + u_2^2)A_\mu A^\mu - \frac{1}{2}\varphi_a\mathscr{H}_{ab}\varphi_b$$

where $\mathscr{H}$ is the Hessian matrix of V at the vacuum

$$\mathscr{H}_{ab} = \left.\frac{\partial^2 V}{\partial|\psi_a||\psi_b|}\right|_{|\psi_1|=u_1, |\psi_2|=u_2}$$

Linearized model

$$\mathcal{L} = \frac{1}{2}\partial_\mu \varphi_a \partial^\mu \varphi_a - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(u_1^2 + u_2^2)A_\mu A^\mu - \frac{1}{2}\varphi_a \mathscr{H}_{ab}\varphi_b$$

Linearized model

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi_a\partial^\mu\varphi_a - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(u_1^2 + u_2^2)A_\mu A^\mu - \frac{1}{2}\varphi_a\mathscr{H}_{ab}\varphi_b$$

- A_μ decouples. In general φ_1, φ_2 do not

Linearized model

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi_a\partial^\mu\varphi_a - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(u_1^2 + u_2^2)A_\mu A^\mu - \frac{1}{2}\varphi_a\mathscr{H}_{ab}\varphi_b$$

- A_μ decouples. In general φ_1, φ_2 do not
- Define eigenvectors v_1, v_2 of $\mathscr{H}$, eigenvalues $\mu_1^2, \mu_2^2 > 0$

Linearized model

$$\mathcal{L} = \frac{1}{2}\partial_\mu \varphi_a \partial^\mu \varphi_a - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(u_1^2 + u_2^2)A_\mu A^\mu - \frac{1}{2}\varphi_a \mathscr{H}_{ab}\varphi_b$$

- A_μ decouples. In general φ_1, φ_2 do not
- Define eigenvectors v_1, v_2 of $\mathscr{H}$, eigenvalues $\mu_1^2, \mu_2^2 > 0$
- Expand

$$\begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix} = \chi_1 v_1 + \chi_2 v_2$$

Linearized model

$$\mathcal{L} = \frac{1}{2}\partial_\mu \varphi_a \partial^\mu \varphi_a - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(u_1^2 + u_2^2)A_\mu A^\mu - \frac{1}{2}\varphi_a \mathscr{H}_{ab}\varphi_b$$

- A_μ decouples. In general φ_1, φ_2 do not
- Define eigenvectors v_1, v_2 of $\mathscr{H}$, eigenvalues $\mu_1^2, \mu_2^2 > 0$
- Expand

$$\begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix} = \chi_1 v_1 + \chi_2 v_2$$

- Defines mixed scalar modes χ_a which decouple

$$\mathcal{L} = \frac{1}{2}\sum_{a=1}^{2}(\partial_\mu \chi_a \partial^\mu \chi_a - \mu_a^2 \chi_a^2) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\mu_A^2 A_\mu A^\mu$$

where $\mu_A = \sqrt{u_1^2 + u_2^2}$ = mass of the photon

Linearized model

- Point vortex carries magnetic dipole moment and two different kinds of scalar monopole charge

Linearized model

- Point vortex carries magnetic dipole moment and two different kinds of scalar monopole charge
- Monopole charges induce scalar fields χ_1, χ_2, mixed fields obtained from φ_1, φ_2 by rotating through mixing angle Θ, where $v_1 = \begin{bmatrix} \cos\Theta \\ \sin\Theta \end{bmatrix}$.

Linearized model

- Point vortex carries magnetic dipole moment and two different kinds of scalar monopole charge
- Monopole charges induce scalar fields χ_1, χ_2, mixed fields obtained from φ_1, φ_2 by rotating through mixing angle Θ, where $v_1 = \begin{bmatrix} \cos\Theta \\ \sin\Theta \end{bmatrix}$.
- Long range attraction $\Leftrightarrow$ $\min\{\mu_1, \mu_2\} < \mu_A$

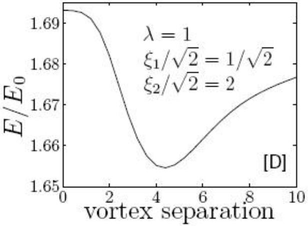
Linearized model

- Point vortex carries magnetic dipole moment and two different kinds of scalar monopole charge
- Monopole charges induce scalar fields χ_1, χ_2, mixed fields obtained from φ_1, φ_2 by rotating through mixing angle Θ, where $v_1 = \begin{bmatrix} \cos\Theta \\ \sin\Theta \end{bmatrix}$.
- Long range attraction $\Leftrightarrow \min\{\mu_1, \mu_2\} < \mu_A$
- Naive expectation: if $\mu_1 < \mu_A < \mu_2$ maybe magnetic repulsion still dominates at short range?

Numerics, basic case (Babaev, JMS)

$$V = \alpha_1|\psi_1|^2 + \frac{\beta_1}{2}|\psi_1|^4 + \alpha_2|\psi_2|^2 + \frac{\beta_2}{2}|\psi_2|^4$$
$$\alpha_1 < 0, \qquad \alpha_2 < 0$$

- VeVs: $u_a = \sqrt{|\alpha_a|/\beta_a}$
- Masses: $\mu_A = \sqrt{u_1^2 + u_2^2}$, $\mu_a = 2\sqrt{|\alpha_a|}$



Type 1.5 superconductivity?

- Imagine we have a superconductor described by previous TCGL and we turn up an applied magnetic field H.

Type 1.5 superconductivity?

- Imagine we have a superconductor described by previous TCGL and we turn up an applied magnetic field H.
- When H reaches $H_{c1} = E(\text{one vortex})/2\pi$ it becomes energetically favourable for magnetic flux to penetrate in a vortex (like type II)

Type 1.5 superconductivity?

- Imagine we have a superconductor described by previous TCGL and we turn up an applied magnetic field H.
- When H reaches $H_{c1} = E(\text{one vortex})/2\pi$ it becomes energetically favourable for magnetic flux to penetrate in a vortex (like type II)
- Increasing H, until we reach $H_{c2} = \mu_0^2 = \max\{2|\alpha_1|, 2|\alpha_2|\}$, more and more vortices penetrate (like type II)...

Type 1.5 superconductivity?

- Imagine we have a superconductor described by previous TCGL and we turn up an applied magnetic field H.
- When H reaches $H_{c1} = E(\text{one vortex})/2\pi$ it becomes energetically favourable for magnetic flux to penetrate in a vortex (like type II)
- Increasing H, until we reach $H_{c2} = \mu_0^2 = \max\{2|\alpha_1|, 2|\alpha_2|\}$, more and more vortices penetrate (like type II)...
- ...**but** it's energetically favourable for the vortices to clump together at a fixed separation, rather than form a regular lattice of increasing density (not like type II)

Type 1.5 superconductivity?

- Imagine we have a superconductor described by previous TCGL and we turn up an applied magnetic field H.
- When H reaches $H_{c1} = E(\text{one vortex})/2\pi$ it becomes energetically favourable for magnetic flux to penetrate in a vortex (like type II)
- Increasing H, until we reach $H_{c2} = \mu_0^2 = \max\{2|\alpha_1|, 2|\alpha_2|\}$, more and more vortices penetrate (like type II)...
- ...**but** it's energetically favourable for the vortices to clump together at a fixed separation, rather than form a regular lattice of increasing density (not like type II)
- Predict clumps of flux penetration in a sea of Meissner state (like type I)...

Type 1.5 superconductivity?

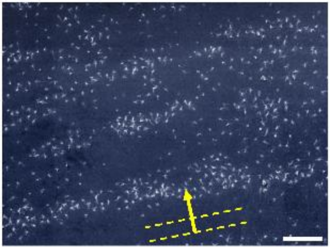
- Imagine we have a superconductor described by previous TCGL and we turn up an applied magnetic field H.
- When H reaches $H_{c1} = E(\text{one vortex})/2\pi$ it becomes energetically favourable for magnetic flux to penetrate in a vortex (like type II)
- Increasing H, until we reach $H_{c2} = \mu_0^2 = \max\{2|\alpha_1|, 2|\alpha_2|\}$, more and more vortices penetrate (like type II)...
- ...**but** it's energetically favourable for the vortices to clump together at a fixed separation, rather than form a regular lattice of increasing density (not like type II)
- Predict clumps of flux penetration in a sea of Meissner state (like type I)...
- ...**but** within each clump, flux will penetrate in a vortex lattice of fixed unit cell size (not like type I)

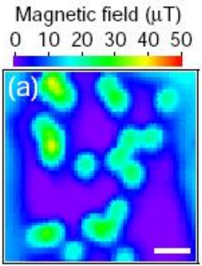
Type 1.5 superconductivity?

- Imagine we have a superconductor described by previous TCGL and we turn up an applied magnetic field H.
- When H reaches $H_{c1} = E(\text{one vortex})/2\pi$ it becomes energetically favourable for magnetic flux to penetrate in a vortex (like type II)
- Increasing H, until we reach $H_{c2} = \mu_0^2 = \max\{2|\alpha_1|, 2|\alpha_2|\}$, more and more vortices penetrate (like type II)...
- ...**but** it's energetically favourable for the vortices to clump together at a fixed separation, rather than form a regular lattice of increasing density (not like type II)
- Predict clumps of flux penetration in a sea of Meissner state (like type I)...
- ...**but** within each clump, flux will penetrate in a vortex lattice of fixed unit cell size (not like type I)
- We called it "semi-Meissner state"...

Type 1.5 superconductivity?

- ...Moshchalkov et al found similar structure in MgB_2





$H = 5$ Oe, Bitter decoration $\quad H = 10\mu T$, SQUID microscopy

- They called it "type 1.5 superconductivity"
- Not universally accepted.

Interband couplings

- Main criticism of our analysis: having no direct coupling between condensates is very unrealistic (only interaction is via em field).

Interband couplings

- Main criticism of our analysis: having no direct coupling between condensates is very unrealistic (only interaction is via em field). In real superconductors, have
  - direct coupling through Josephson effect

$$V_{Jos} = -\frac{\eta_1}{2}(\overline{\psi_1}\psi_2 + \psi_1\overline{\psi_2})$$

Interband couplings

- Main criticism of our analysis: having no direct coupling between condensates is very unrealistic (only interaction is via em field). In real superconductors, have
  - direct coupling through Josephson effect

$$V_{Jos} = -\frac{\eta_1}{2}(\overline{\psi_1}\psi_2 + \psi_1\overline{\psi_2})$$

  - gradient-gradient coupling (except in ultra clean samples) due to electron scattering off impurities

$$\nu Re(\mathbf{D}\psi_1 \cdot \overline{\mathbf{D}\psi_2})$$

Interband couplings

- Main criticism of our analysis: having no direct coupling between condensates is very unrealistic (only interaction is via em field). In real superconductors, have
  - direct coupling through Josephson effect

$$V_{Jos} = -\frac{\eta_1}{2}(\overline{\psi_1}\psi_2 + \psi_1\overline{\psi_2})$$

  - gradient-gradient coupling (except in ultra clean samples) due to electron scattering off impurities

$$\nu Re(\mathbf{D}\psi_1 \cdot \overline{\mathbf{D}\psi_2})$$

  - Also, if we're including terms up to order 4, why don't we include

$$V_{Quartic} = \eta_2|\psi_1|^2|\psi_2|^2?$$

Interband couplings

- Main criticism of our analysis: having no direct coupling between condensates is very unrealistic (only interaction is via em field). In real superconductors, have
  - direct coupling through Josephson effect

$$V_{Jos} = -\frac{\eta_1}{2}(\overline{\psi_1}\psi_2 + \psi_1\overline{\psi_2})$$

  - gradient-gradient coupling (except in ultra clean samples) due to electron scattering off impurities

$$\nu Re(\mathbf{D}\psi_1 \cdot \overline{\mathbf{D}\psi_2})$$

  - Also, if we're including terms up to order 4, why don't we include

$$V_{Quartic} = \eta_2|\psi_1|^2|\psi_2|^2?$$

- Once condensates are coupled, expect this to equalize their decay rates as $r \to \infty$. Maybe this eliminates the type 1.5 regime altogether?

Interband couplings

- Riposte: direct coupling terms are forbidden in many interesting systems (e.g. liquid metallic hydrogen), so our original analysis is still relevant to such systems.

Interband couplings

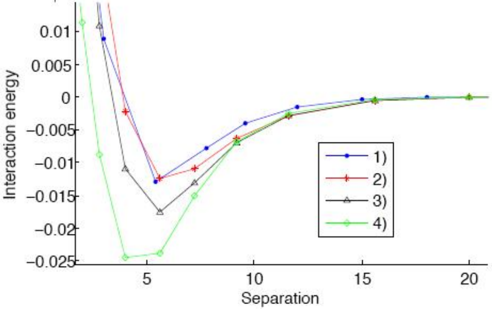
- Riposte: direct coupling terms are forbidden in many interesting systems (e.g. liquid metallic hydrogen), so our original analysis is still relevant to such systems.
- Better riposte: the length scales of interest are inverse masses of the (now mixed) normal modes **not** the condensates themselves. Can still have splitting $\mu_1 < \mu_A < \mu_2$

Interband couplings

- Riposte: direct coupling terms are forbidden in many interesting systems (e.g. liquid metallic hydrogen), so our original analysis is still relevant to such systems.
- Better riposte: the length scales of interest are inverse masses of the (now mixed) normal modes **not** the condensates themselves. Can still have splitting $\mu_1 < \mu_A < \mu_2$
- Even better riposte: large scale numerical simulations of the model including all these extra terms show that there are big regions of parameter space where vortex interaction is non-monotonic [Babaev, Carlstrom, JMS].

Interband couplings

Curve	α_1	β_1	α_2	β_2	η_1	η_2	e	$\langle\lvert\psi_1\rvert^2\rangle$	$\langle\lvert\psi_2\rvert^2\rangle$
1	-1	1	-0.0069	0.0278	0.0222	0	0.7	1.0094	0.723
2	-1	1	-0.0069	0.0278	0	-0.0077	0.7	1.004	0.526
3	-1	1	-0.0069	0.0278	0	-0.013	0.7	1.0094	0.72
4	-1	1	-0.0069	0.0278	0	-0.0232	0.7	1.026	1.1



- Doesn't answer question of whether *MgB*$_2$ supports type 1.5 superconductivity (have no idea what the interband coupling parameters are). But it does show that type 1.5 superconductivity is possible in principle.

Summary

- In two component superconductors there can be thermodynamically stable vortices with attractive interaction at long range.

Summary

- In two component superconductors there can be thermodynamically stable vortices with attractive interaction at long range.
- This behaviour is robust and survives all kinds of interband coupling.

Summary

- In two component superconductors there can be thermodynamically stable vortices with attractive interaction at long range.
- This behaviour is robust and survives all kinds of interband coupling.
- Modes mediating long range attraction are mixed.

Summary

- In two component superconductors there can be thermodynamically stable vortices with attractive interaction at long range.
- This behaviour is robust and survives all kinds of interband coupling.
- Modes mediating long range attraction are mixed.
- Leads to appearance of "semi-Meissner" state.

Summary

- In two component superconductors there can be thermodynamically stable vortices with attractive interaction at long range.
- This behaviour is robust and survives all kinds of interband coupling.
- Modes mediating long range attraction are mixed.
- Leads to appearance of "semi-Meissner" state.
- Can apply to two-band materials, metallic hydrogen, maybe even neutron stars...