Vortex lattices in anisotropic superconductors.

Martin Speight (Leeds)
joint with
Thomas Winyard (Edinburgh)

SIG XII 25/6/24

Based on arXiv:2406.16584

Vortices in superconductors

$$F = \int_\Omega \frac{1}{2}|D_i\psi|^2 + \alpha(T)|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2}|B|^2$$

Vortices in superconductors

$$F = \int_\Omega \frac{1}{2}|D_i\psi|^2 - \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2}|B|^2$$

Vortices in superconductors

$$F = \int_\Omega \frac{1}{2}|D_i\psi|^2 - \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{\alpha^2}{2\beta} + \frac{1}{2}|B|^2$$

Vortices in superconductors

$$F = \int_\Omega \frac{1}{2}|D_i\psi|^2 + \frac{\lambda}{8}(1 - |\psi|^2)^2 + \frac{1}{2}|B|^2$$

Vortices in superconductors

$$F = \int_{\Omega} \frac{1}{2}|D_i\psi|^2 + \frac{\lambda}{8}(1 - |\psi|^2)^2 + \frac{1}{2}|B|^2$$

- Applied field H

Vortices in superconductors

$$F = \int_\Omega \frac{1}{2}|D_i\psi|^2 + \frac{\lambda}{8}(1-|\psi|^2)^2 + \frac{1}{2}|B|^2$$

- Applied field H

$$G = \int_\Omega \frac{1}{2}|D_i\psi|^2 + \frac{\lambda}{8}(1-|\psi|^2)^2 + \frac{1}{2}|B-H|^2$$

Vortices in superconductors

$$F = \int_\Omega \frac{1}{2}|D_i\psi|^2 + \frac{\lambda}{8}(1 - |\psi|^2)^2 + \frac{1}{2}|B|^2$$

- Applied field H

$$\begin{aligned} G &= \int_\Omega \frac{1}{2}|D_i\psi|^2 + \frac{\lambda}{8}(1 - |\psi|^2)^2 + \frac{1}{2}|B - H|^2 \\ &= F - \langle B, H\rangle_{L^2(\Omega)} + \frac{1}{2}\int_\Omega |H|^2 \end{aligned}$$

Vortices in superconductors

$$F = \int_\Omega \frac{1}{2}|D_i\psi|^2 + \frac{\lambda}{8}(1-|\psi|^2)^2 + \frac{1}{2}|B|^2$$

- Applied field H

$$\begin{aligned} G &= \int_\Omega \frac{1}{2}|D_i\psi|^2 + \frac{\lambda}{8}(1-|\psi|^2)^2 + \frac{1}{2}|B-H|^2 \\ &= F - \langle B, H\rangle_{L^2(\Omega)} + \frac{1}{2}\int_\Omega |H|^2 \end{aligned}$$

- Same critical points. **Stability** depends on H

Vortices in superconductors

- Normal state: $B = H$, $\psi = 0$, $\langle G \rangle_{NS} = \frac{\lambda}{8}$

Vortices in superconductors

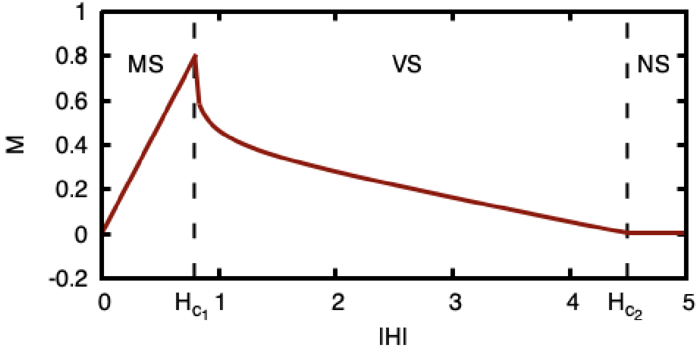
- Normal state: $B = H$, $\psi = 0$, $\langle G \rangle_{NS} = \frac{\lambda}{8}$
- Meißner state: $B = 0$, $|\psi| = 1$, $\langle G \rangle_{MS} = \frac{1}{2}|H|^2$.

Vortices in superconductors

- Normal state: $B = H$, $\psi = 0$, $\langle G \rangle_{NS} = \frac{\lambda}{8}$
- Meißner state: $B = 0$, $|\psi| = 1$, $\langle G \rangle_{MS} = \frac{1}{2}|H|^2$.
- Thermodynamic critical field $H_c = \frac{\sqrt{\lambda}}{2}$

Vortices in superconductors

- Normal state: $B = H$, $\psi = 0$, $\langle G \rangle_{NS} = \frac{\lambda}{8}$
- Meißner state: $B = 0$, $|\psi| = 1$, $\langle G \rangle_{MS} = \frac{1}{2}|H|^2$.
- Thermodynamic critical field $H_c = \frac{\sqrt{\lambda}}{2}$



Critical fields

- $F_1 :=$ Helmholtz energy of a single vortex:

$$G_1 - G_{MS} = F_1 - \langle B, H \rangle + \frac{1}{2}\|H\|^2 - \frac{1}{2}\|H\|^2 = F_1 - 2\pi|H|$$

Critical fields

- $F_1 :=$ Helmholtz energy of a single vortex:

$$G_1 - G_{MS} = F_1 - \langle B, H \rangle + \frac{1}{2}\|H\|^2 - \frac{1}{2}\|H\|^2 = F_1 - 2\pi|H|$$

- Lower critical field $H_{c1} = \frac{F_1}{2\pi}$

Critical fields

- F_1 := Helmholtz energy of a single vortex:

$$G_1 - G_{MS} = F_1 - \langle B, H \rangle + \frac{1}{2}\|H\|^2 - \frac{1}{2}\|H\|^2 = F_1 - 2\pi|H|$$

- Lower critical field $H_{c1} = \frac{F_1}{2\pi}$
- Upper critical field: H_{c2} := applied field above which NS is linearly stable

Critical fields

- $F_1 :=$ Helmholtz energy of a single vortex:

$$G_1 - G_{MS} = F_1 - \langle B, H \rangle + \frac{1}{2}\|H\|^2 - \frac{1}{2}\|H\|^2 = F_1 - 2\pi|H|$$

- Lower critical field $H_{c1} = \frac{F_1}{2\pi}$
- Upper critical field: $H_{c2} :=$ applied field above which NS is linearly stable

$$H_{c2} = \frac{\lambda}{2}$$

Critical fields: type I ($\lambda < 1$)

$$H_{c1} = \frac{F_1}{2\pi}, \qquad H_{c2} = \frac{\lambda}{2}$$

$$F_1 = \int_\Omega \frac{1}{2}|D\psi|^2 + \frac{1}{2}|B|^2 + \frac{\lambda}{8}(1 - |\psi|^2)^2$$

Critical fields: type I ($\lambda < 1$)

$$H_{c1} = \frac{F_1}{2\pi}, \qquad H_{c2} = \frac{\lambda}{2}$$

$$\begin{aligned} F_1 &= \int_\Omega \frac{1}{2}|D\psi|^2 + \frac{1}{2}|B|^2 + \frac{\lambda}{8}(1-|\psi|^2)^2 \\ &> \lambda \int_\Omega \frac{1}{2}|D\psi|^2 + \frac{1}{2}|B|^2 + \frac{1}{8}(1-|\psi|^2)^2 \end{aligned}$$

Critical fields: type I ($\lambda < 1$)

$$H_{c1} = \frac{F_1}{2\pi}, \qquad H_{c2} = \frac{\lambda}{2}$$

$$\begin{aligned} F_1 &= \int_\Omega \frac{1}{2}|D\psi|^2 + \frac{1}{2}|B|^2 + \frac{\lambda}{8}(1-|\psi|^2)^2 \\ &> \lambda \int_\Omega \frac{1}{2}|D\psi|^2 + \frac{1}{2}|B|^2 + \frac{1}{8}(1-|\psi|^2)^2 \\ &\geq \lambda\pi \end{aligned}$$

Critical fields: type I ($\lambda < 1$)

$$H_{c1} = \frac{F_1}{2\pi}, \qquad H_{c2} = \frac{\lambda}{2}$$

$$\begin{aligned} F_1 &= \int_\Omega \frac{1}{2}|D\psi|^2 + \frac{1}{2}|B|^2 + \frac{\lambda}{8}(1 - |\psi|^2)^2 \\ &> \lambda \int_\Omega \frac{1}{2}|D\psi|^2 + \frac{1}{2}|B|^2 + \frac{1}{8}(1 - |\psi|^2)^2 \\ &\geq \lambda\pi \end{aligned}$$

Hence $H_{c1} > H_{c2}$. **No vortices!**

Critical fields: type II ($\lambda > 1$)

$$H_{c1} = \frac{F_1}{2\pi}, \qquad H_{c2} = \frac{\lambda}{2}$$

$$F_1 = \int_\Omega \frac{1}{2}|D\psi|^2 + \frac{1}{2}|B|^2 + \frac{\lambda}{8}(1 - |\psi|^2)^2$$

Critical fields: type II ($\lambda > 1$)

$$H_{c1} = \frac{F_1}{2\pi}, \qquad H_{c2} = \frac{\lambda}{2}$$

$$\begin{aligned} F_1 &= \int_\Omega \frac{1}{2}|D\psi|^2 + \frac{1}{2}|B|^2 + \frac{\lambda}{8}(1-|\psi|^2)^2 \\ &< \lambda \int_\Omega \frac{1}{2}|D\psi|^2 + \frac{1}{2}|B|^2 + \frac{1}{8}(1-|\psi|^2)^2 \end{aligned}$$

Critical fields: type II ($\lambda > 1$)

$$H_{c1} = \frac{F_1}{2\pi}, \qquad H_{c2} = \frac{\lambda}{2}$$

$$\begin{aligned} F_1 &= \int_\Omega \frac{1}{2}|D\psi|^2 + \frac{1}{2}|B|^2 + \frac{\lambda}{8}(1 - |\psi|^2)^2 \\ &< \lambda \int_\Omega \frac{1}{2}|D\psi|^2 + \frac{1}{2}|B|^2 + \frac{1}{8}(1 - |\psi|^2)^2 \\ &= \lambda\pi \end{aligned}$$

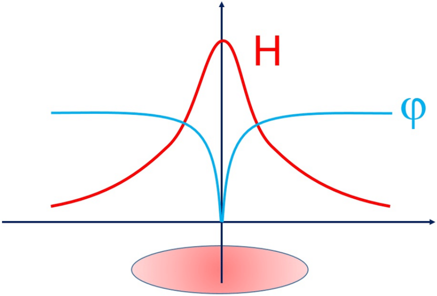
Critical fields: type II ($\lambda > 1$)

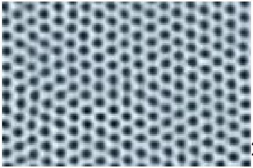
$$H_{c1} = \frac{F_1}{2\pi}, \qquad H_{c2} = \frac{\lambda}{2}$$

$$\begin{aligned} F_1 &= \int_\Omega \frac{1}{2}|D\psi|^2 + \frac{1}{2}|B|^2 + \frac{\lambda}{8}(1-|\psi|^2)^2 \\ &< \lambda \int_\Omega \frac{1}{2}|D\psi|^2 + \frac{1}{2}|B|^2 + \frac{1}{8}(1-|\psi|^2)^2 \\ &= \lambda\pi \end{aligned}$$

Hence $H_{c1} < H_{c2}$. **Vortices!**

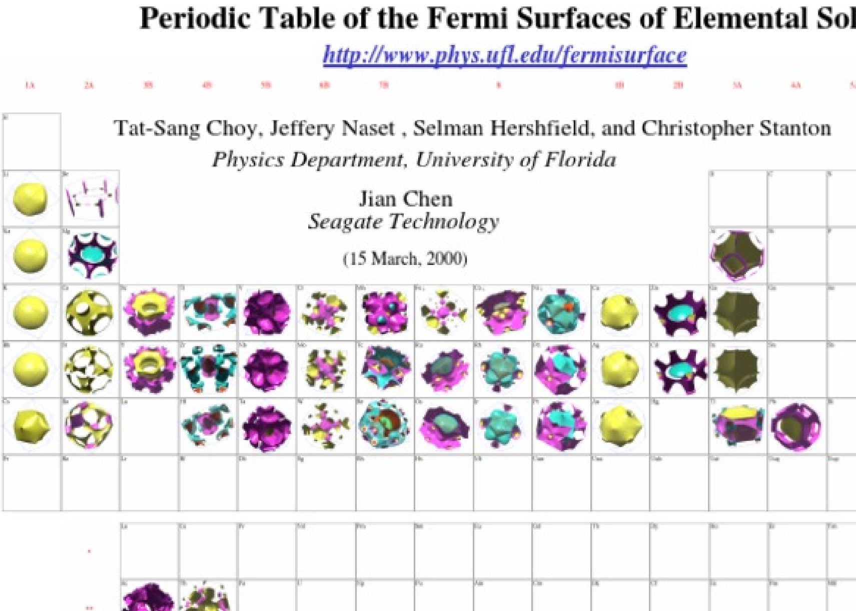
Abrikosov lattice



1

2

[1]Picture credit: H. Suganuma , Y. Nakagawa , K. Matsumoto (Kyoto)
[2]Picture credit: Somesh Chandra Ganguli (Aalto Un., Helsinki)

Fermi surface



Fermi surface

- Far from isotropic
- Multiple bands
- exotic pairing possible:
  - spin triplet p-wave
  - spin singlet d-wave
  - mix and match
- Multicomponent, anisotropic GL model

Multicomponent anisotropic GL theory

- Several condensates ψ_α, $\alpha = 1, 2, \ldots, N$.

$$F = \frac{1}{2} Q_{ij}^{\alpha\beta} \overline{D_i \psi_\alpha} D_j \psi_\beta + V(\psi) + \frac{1}{2}|B|^2$$

- $Q_{ij}^{\alpha\beta} = \bar{Q}_{ji}^{\beta\alpha}$
- $V(e^{i\theta}\psi) = V(\psi)$

$$\begin{aligned} -Q_{ij}^{\alpha\beta} D_i D_j \psi_\beta + 2\frac{\partial V}{\partial \bar{\psi}_\alpha} &= 0 \\ -\partial_j(\partial_j A_i - \partial_i A_j) &= \mathrm{Im}\left(Q_{ij}^{\alpha\beta} \bar{\psi}_\alpha D_j \psi_\beta\right) \end{aligned}$$

Flux quantization

$$F = \int_{\mathbb{R}^2} \frac{1}{2} Q(D\psi, D\psi) + V(\psi) + \frac{1}{2}|B|^2$$

- $V \geq 0, \; V(u) = 0, \; u \neq 0$
- As $r \to \infty$, $[\psi] \to [u]$, $D\psi \to 0$

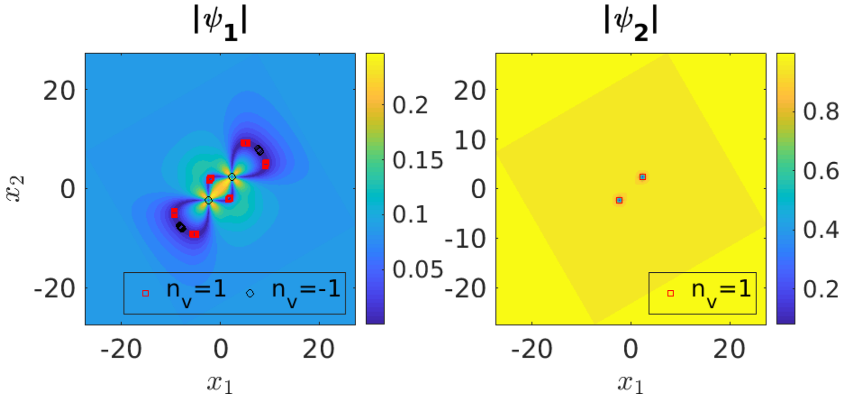
$$\psi \sim u e^{i\chi(\theta)}, \qquad A \sim d\chi$$

- Flux quantization

$$\int_{\mathbb{R}^2} B = \oint_{S^1_\infty} A = \chi(2\pi) - \chi(0) = 2\pi n$$

- Each ψ_α has n zeroes (counted with multiplicity)

It's complicated[3]



[3] Zhang et al Phys. Rev. B 101, 064501 (2020)

Vortex lattices

A **vortex lattice** is a solution of the Euler-Lagrange equations which is

Vortex lattices

A **vortex lattice** is a solution of the Euler-Lagrange equations which is

- translation invariant in some direction $v_3 \in \mathbb{R}^3$

Vortex lattices

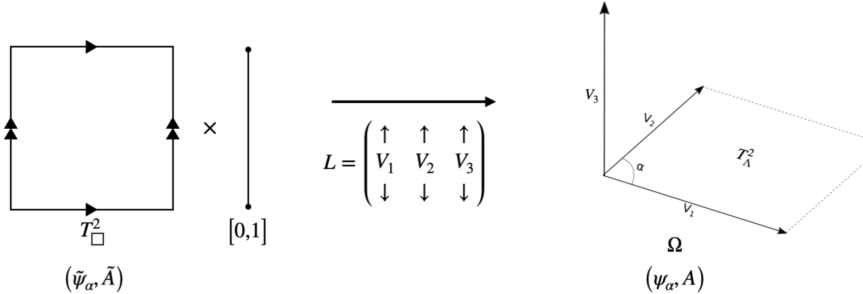
A **vortex lattice** is a solution of the Euler-Lagrange equations which is

- translation invariant in some direction $v_3 \in \mathbb{R}^3$
- periodic with respect to some lattice $v_1, v_2 \perp v_3$

Vortex lattices

A **vortex lattice** is a solution of the Euler-Lagrange equations which is

- translation invariant in some direction $v_3 \in \mathbb{R}^3$
- periodic with respect to some lattice $v_1, v_2 \perp v_3$



Vortex lattices

Work with fields $\widetilde{\psi}_\alpha(X_1, X_2)$, $\widetilde{A}_i(X_1, X_2)$, $i = 1, 2, 3$ on unit square $[0,1]^2$ with

$$\begin{aligned}
\widetilde{\psi}_\alpha(X_1+1, X_2) &= \widetilde{\psi}_\alpha(X_1, X_2) e^{2\pi i n X_2} \\
\widetilde{\psi}_\alpha(X_1, X_2+1) &= \widetilde{\psi}_\alpha(X_1, X_2) \\
\widetilde{A}_1(X_1+1, X_2) &= \widetilde{A}_1(X_1, X_2) \\
\widetilde{A}_2(X_1+1, X_2) &= \widetilde{A}_2(X_1, X_2) + 2\pi n \\
\widetilde{A}_3(X_1+1, X_2) &= \widetilde{A}_3(X_1, X_2) \\
\widetilde{A}_i(X_1, X_2+1) &= \widetilde{A}_i(X_1, X_2)
\end{aligned}$$

Optimal lattice geometry?

Should minimize

$$
\begin{aligned}
G &= \int_\Omega \left\{ \frac{1}{2} Q(D\psi, D\psi) + V(\psi) + \frac{1}{2}|B - H|^2 \right\} \\
&= F - H \cdot \int_\Omega B + \frac{1}{2} \int_\Omega |H|^2
\end{aligned}
$$

w.r.t. ψ_α, A **and** L (**and** $n = \deg \mathcal{L}$):

Optimal lattice geometry?

Should minimize

$$\begin{aligned} G &= \int_\Omega \left\{ \frac{1}{2} Q(D\psi, D\psi) + V(\psi) + \frac{1}{2}|B - H|^2 \right\} \\ &= F - H \cdot \int_\Omega B + \frac{1}{2} \int_\Omega |H|^2 \end{aligned}$$

w.r.t. ψ_α, A **and** L (**and** $n = \deg \mathcal{L}$):

$$G = \frac{1}{2} L^{-1}_{ki} P_{ki,lj} L^{-1}_{lj} + \frac{1}{2} \mathrm{tr}\,(L\mathbb{F}L^T) - 2n\pi H_i L_{i3} + \int_{T^2 \times [0,1]} V(\psi),$$

where

$$\begin{aligned} P_{ki,lj} &= \mathrm{Re} \int_{T^2 \times [0,1]} Q^{\alpha\beta}_{ij} \overline{D_k \psi_\alpha} D_l \psi_\beta \\ \mathbb{F}_{ij} &= \int_{T^2 \times [0,1]} B_i B_j. \end{aligned}$$

Numerical method

- Discretize unit square

$$G : \mathbb{R}^{(2N+3)N_1 N_2} \times \mathscr{C} \to \mathbb{R}$$

where

$$\mathscr{C} = \{L \in GL(3, \mathbb{R}) : \det L = 1,\ L_{i1} L_{i3} = 0,\ L_{i2} L_{i3} = 0\} \subset \mathbb{R}^9$$

Numerical method

- Discretize unit square

$$G : \mathbb{R}^{(2N+3)N_1 N_2} \times \mathscr{C} \to \mathbb{R}$$

  where

$$\mathscr{C} = \{L \in GL(3, \mathbb{R}) : \det L = 1,\ L_{i1} L_{i3} = 0,\ L_{i2} L_{i3} = 0\} \subset \mathbb{R}^9$$

- Minimize a function on a (codimension 3 submfd of) a big Euclidean space.

Numerical method

- Discretize unit square

$$G : \mathbb{R}^{(2N+3)N_1 N_2} \times \mathscr{C} \to \mathbb{R}$$

  where

$$\mathscr{C} = \{L \in GL(3, \mathbb{R}) : \det L = 1,\ L_{i1} L_{i3} = 0,\ L_{i2} L_{i3} = 0\} \subset \mathbb{R}^9$$

- Minimize a function on a (codimension 3 submfd of) a big Euclidean space.
- Arrested Newton flow.

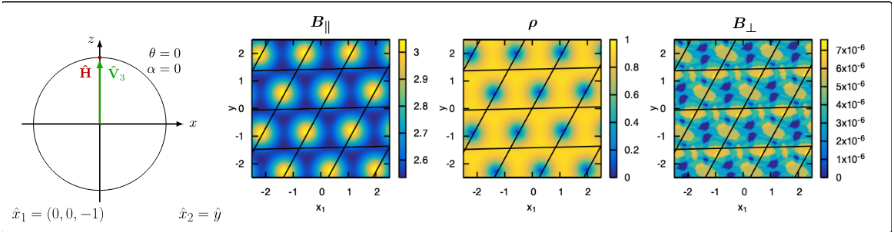
Vortex line tilting

- **Single** component model:

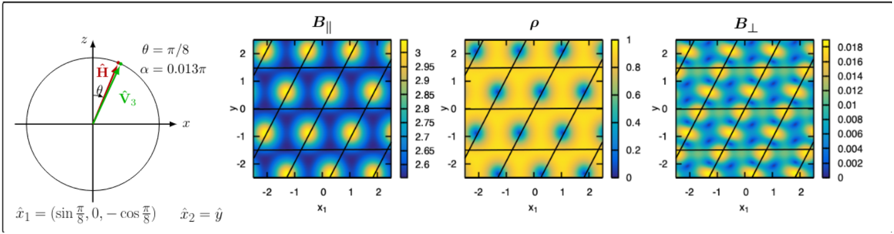
$$Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.1 \end{pmatrix}, \qquad V = \frac{9}{4}(1 - |\psi|^2)^2$$

- Optimal lattice does **not** have $\mathbf{v}_3 \parallel H$ if H not an eigenvector of Q

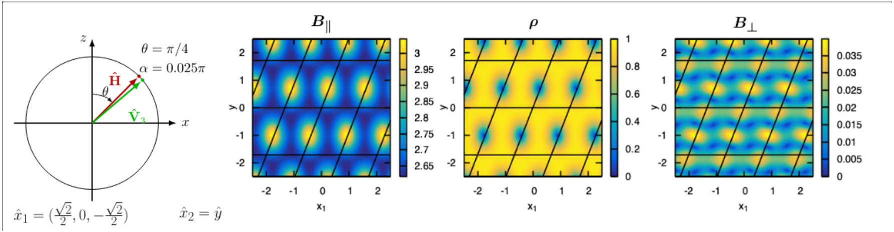
Vortex line tilting



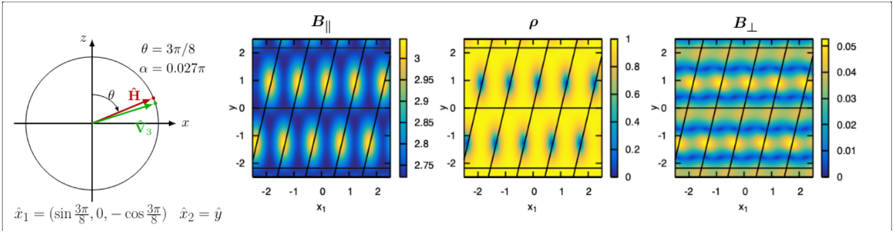
Vortex line tilting



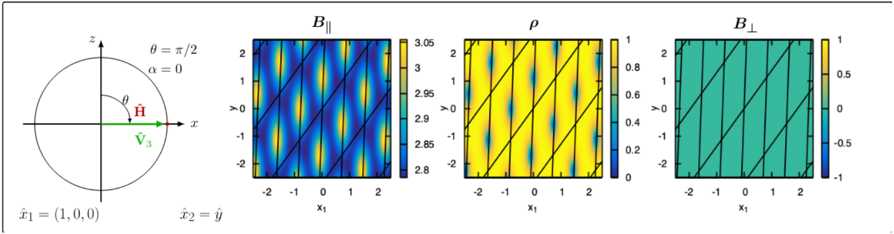
Vortex line tilting



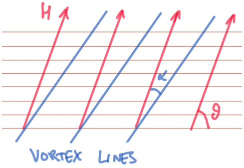
Vortex line tilting

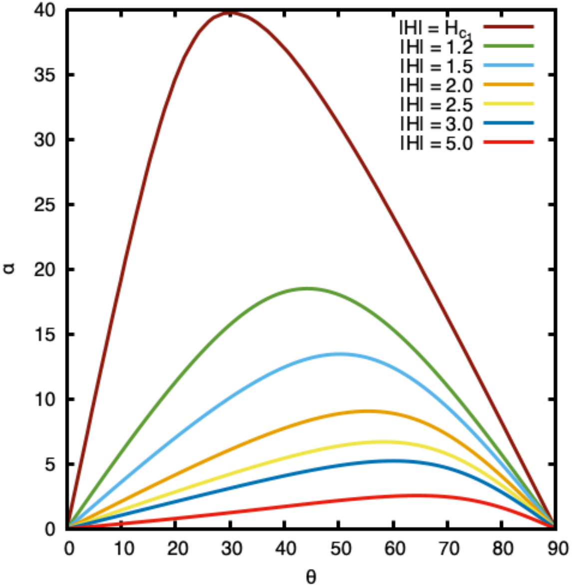


Vortex line tilting



Vortex line tilting





Finding H_{c1}

- Fix $\hat{H} \in S^2$. Start with $|H|$ small.

Finding H_{c1}

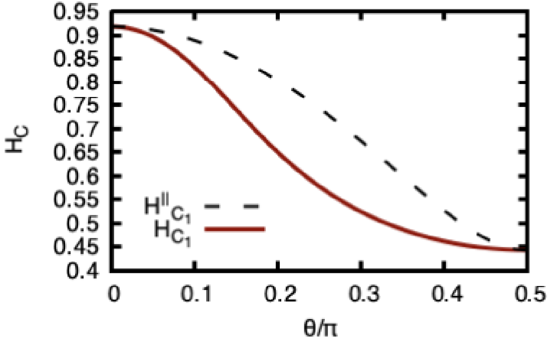
- Fix $\hat{H} \in S^2$. Start with $|H|$ small.
- Run lattice minimization algorithm with $|v_1| = |v_2| = L$, large fixed and $v_1 \cdot v_2 = 0$.

Finding H_{c1}

- Fix $\hat{H} \in S^2$. Start with $|H|$ small.
- Run lattice minimization algorithm with $|v_1| = |v_2| = L$, large fixed and $v_1 \cdot v_2 = 0$.
- Increment $|H|$ until G crosses below $G_{Meißner}$

Finding H_{c1}

- Fix $\hat{H} \in S^2$. Start with $|H|$ small.
- Run lattice minimization algorithm with $|v_1| = |v_2| = L$, large fixed and $v_1 \cdot v_2 = 0$.
- Increment $|H|$ until G crosses below $G_{Meißner}$



Finding H_{c2}

- Second variation of G about normal state $\psi = 0$, $\mathrm{d}A = H$

$$\frac{d^2}{dt^2}\bigg|_{t=0} G(\psi_t, A_t) = \langle \dot{\psi}_\alpha, -Q_{ij}^{\alpha\beta} D_i D_j \dot{\psi}_\beta + M_{\alpha\beta}\dot{\psi}_\beta \rangle_{L^2(\Omega)} + \|\mathrm{d}\dot{A}\|^2_{L^2(\Omega)}$$

where $M_{\alpha\beta} = 2\partial^2 V / \partial\overline{\psi}_\alpha \partial\psi_\beta|_0$

Finding H_{c2}

- Second variation of G about normal state $\psi = 0$, $\mathrm{d}A = H$

$$\frac{d^2}{dt^2}\bigg|_{t=0} G(\psi_t, A_t) = \langle \dot{\psi}_\alpha, -Q_{ij}^{\alpha\beta} D_i D_j \dot{\psi}_\beta + M_{\alpha\beta}\dot{\psi}_\beta \rangle_{L^2(\Omega)} + \|\mathrm{d}\dot{A}\|^2_{L^2(\Omega)}$$

  where $M_{\alpha\beta} = 2\partial^2 V/\partial\overline{\psi}_\alpha \partial\psi_\beta|_0$

- Normal state stable $\Leftrightarrow$

$$\widehat{O} = -Q_{ij} D_i D_j + M$$

  has positive spectrum.

Finding H_{c2}

- Second variation of G about normal state $\psi = 0$, $\mathrm{d}A = H$

$$\left.\frac{d^2}{dt^2}\right|_{t=0} G(\psi_t, A_t) = \langle \dot{\psi}_\alpha, -Q_{ij}^{\alpha\beta} D_i D_j \dot{\psi}_\beta + M_{\alpha\beta}\dot{\psi}_\beta \rangle_{L^2(\Omega)} + \|\mathrm{d}\dot{A}\|^2_{L^2(\Omega)}$$

  where $M_{\alpha\beta} = 2\partial^2 V / \partial\overline{\psi}_\alpha \partial\psi_\beta|_0$

- Normal state stable $\Leftrightarrow$

$$\widehat{O} = -Q_{ij} D_i D_j + M$$

  has positive spectrum.

- Rotate coordinate system so that $H = (0, 0, |H|)$

$$R = \begin{pmatrix} \updownarrow & \updownarrow & \widehat{H} \end{pmatrix}, \qquad Q \mapsto R^T Q R$$

Finding H_{c2}

- Rescale spatial coords $Y_i = \sqrt{|H|/2}X_i$

$$\widehat{O} = -\frac{|H|}{2} Q_{ij} \mathscr{D}_i \mathscr{D}_j + M$$

$$\mathscr{D}_1 = \partial_{Y_1} + iY_2,\ \mathscr{D}_2 = \partial_{Y_2} - iY_1,\ \mathscr{D}_3 = \partial_{Y_3}.$$

Finding H_{c2}

- Rescale spatial coords $Y_i = \sqrt{|H|/2} X_i$

$$\widehat{O} = -\frac{|H|}{2} Q_{ij} \mathscr{D}_i \mathscr{D}_j + M$$

$$\mathscr{D}_1 = \partial_{Y_1} + iY_2,\ \mathscr{D}_2 = \partial_{Y_2} - iY_1,\ \mathscr{D}_3 = \partial_{Y_3}.$$

- $[\widehat{O}, i\mathscr{D}_3] = 0$, seek simultaneous eigenstates

$$\dot{\psi} = \phi(Y_1, Y_2) e^{ikY_3}$$

Finding H_{c2}

- Rescale spatial coords $Y_i = \sqrt{|H|/2}X_i$

$$\widehat{O} = -\frac{|H|}{2} Q_{ij}\mathscr{D}_i\mathscr{D}_j + M$$

$$\mathscr{D}_1 = \partial_{Y_1} + iY_2,\ \mathscr{D}_2 = \partial_{Y_2} - iY_1,\ \mathscr{D}_3 = \partial_{Y_3}.$$

- $[\widehat{O}, i\mathscr{D}_3] = 0$, seek simultaneous eigenstates

$$\dot{\psi} = \phi(Y_1, Y_2)e^{ikY_3}$$

- $a = \frac{i}{2}(\mathscr{D}_1 + i\mathscr{D}_2)$, $a^\dagger = \frac{i}{2}(\mathscr{D}_1 - i\mathscr{D}_2)$ satisfy the harmonic oscillator algebra

$$[a, a^\dagger] = 1$$

Finding H_{c2}

- Rescale spatial coords $Y_i = \sqrt{|H|/2}X_i$

$$\hat{O} = -\frac{|H|}{2}Q_{ij}\mathscr{D}_i\mathscr{D}_j + M$$

$$\mathscr{D}_1 = \partial_{Y_1} + iY_2,\ \mathscr{D}_2 = \partial_{Y_2} - iY_1,\ \mathscr{D}_3 = \partial_{Y_3}.$$

- $[\hat{O}, i\mathscr{D}_3] = 0$, seek simultaneous eigenstates

$$\dot{\psi} = \phi(Y_1, Y_2)e^{ikY_3}$$

- $a = \frac{i}{2}(\mathscr{D}_1 + i\mathscr{D}_2)$, $a^\dagger = \frac{i}{2}(\mathscr{D}_1 - i\mathscr{D}_2)$ satisfy the harmonic oscillator algebra

$$[a, a^\dagger] = 1$$

- $\hat{O}$ reduces to (infinite) tridiagonal matrix acting on "particle states" $|m\rangle = (a^\dagger)^m|0\rangle$, $|0\rangle = e^{-(Y_1^2+Y_2^2)/2}$.

Finding H_{c2}

- Single component, isotropic

$$Q = \mathbb{I}_3, \qquad V = \frac{\lambda}{8}(1 - |\psi|^2)^2$$

Finding H_{c2}

- Single component, isotropic

$$Q = \mathbb{I}_3, \qquad V = \frac{\lambda}{8}(1 - |\psi|^2)^2$$

$$\widehat{O}_k = |H|(a^\dagger a + aa^\dagger) + k^2 - \frac{\lambda}{2}$$

- Clearly ground state has $k = 0$, $\phi = |0\rangle$:

$$E_0 = |H| - \frac{\lambda}{2} \qquad \Rightarrow \qquad H_{c2} = \frac{\lambda}{2}.$$

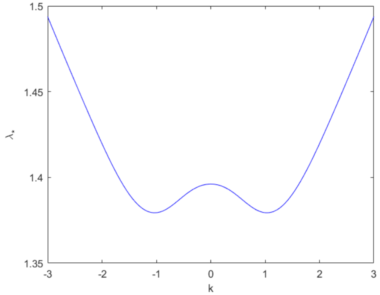
Finding H_{c2}

- No reason in general why ground state should have $k = 0$

Finding H_{c2}

- No reason in general why ground state should have $k = 0$

$$Q^{11} = Q^{22} = \mathbb{I}_3, \quad Q^{12} = \begin{pmatrix} -0.35 & -0.25 & 0.39 \\ -0.24 & 0.11 & 0.38 \\ 0.42 & 0.37 & -0.4 \end{pmatrix} + i \begin{pmatrix} 0.11 & 0.21 & 0.27 \\ 0 & -0.1 & 0.07 \\ 0.18 & 0.14 & 0.22 \end{pmatrix}$$



An $s + id$ model

$$Q^{11} = \frac{1}{\sqrt{2}}\,\mathrm{diag}(1, 1, 0.1), \qquad Q^{22} = \frac{1}{2} Q^{11}, \qquad Q^{12} = \frac{1}{2\sqrt{2}}\mathrm{diag}(1, -1, 0)$$

An $s + id$ model

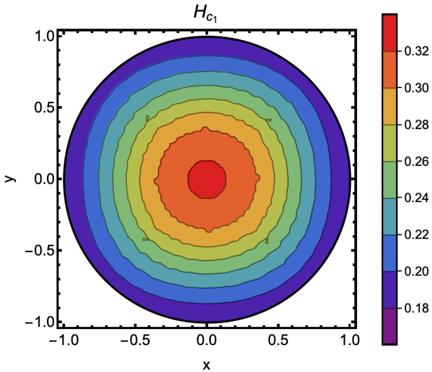
$$Q^{11} = \frac{1}{\sqrt{2}}\,\mathrm{diag}(1, 1, 0.1), \qquad Q^{22} = \frac{1}{2} Q^{11}, \qquad Q^{12} = \frac{1}{2\sqrt{2}}\mathrm{diag}(1, -1, 0)$$

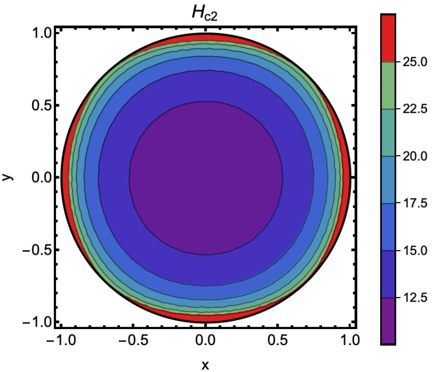
$$V = -1.4|\psi_1|^2 - |\psi_2|^2 + \frac{2}{3}|\psi_1|^4 + \frac{1}{4}|\psi_2|^4 + \frac{8}{3}|\psi_1|^2|\psi_2|^2 + \frac{2}{3}(\psi_1^2\overline{\psi}_2^2 + \overline{\psi}_1^2\psi_2^2)$$

An $s + id$ model

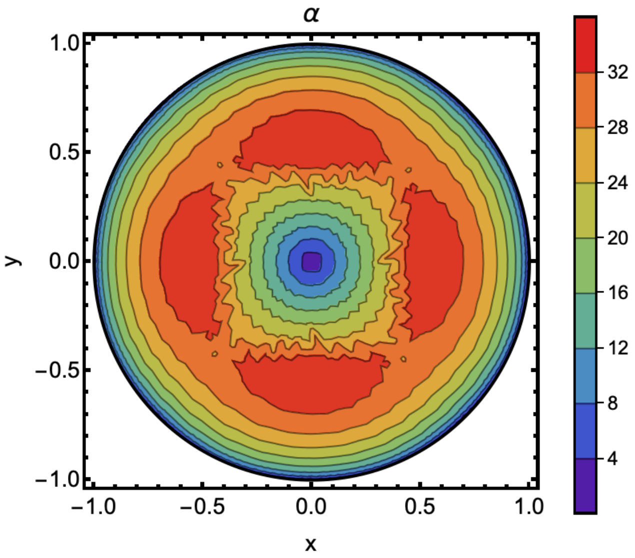
$$Q^{11} = \frac{1}{\sqrt{2}}\,\mathrm{diag}(1, 1, 0.1), \qquad Q^{22} = \frac{1}{2}Q^{11}, \qquad Q^{12} = \frac{1}{2\sqrt{2}}\mathrm{diag}(1, -1, 0)$$

$$V = -1.4|\psi_1|^2 - |\psi_2|^2 + \frac{2}{3}|\psi_1|^4 + \frac{1}{4}|\psi_2|^4 + \frac{8}{3}|\psi_1|^2|\psi_2|^2 + \frac{2}{3}(\psi_1^2\overline{\psi}_2^2 + \overline{\psi}_1^2\psi_2^2)$$

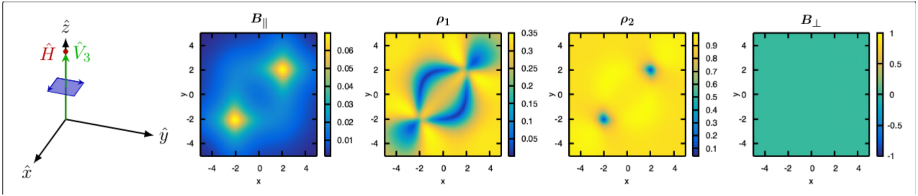




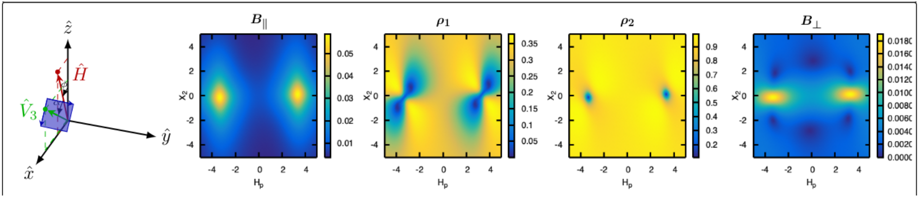
An $s + id$ model: vortex line tilting



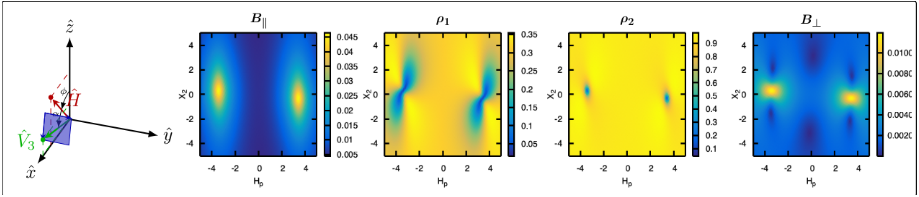
An $s + id$ model: vortex splitting



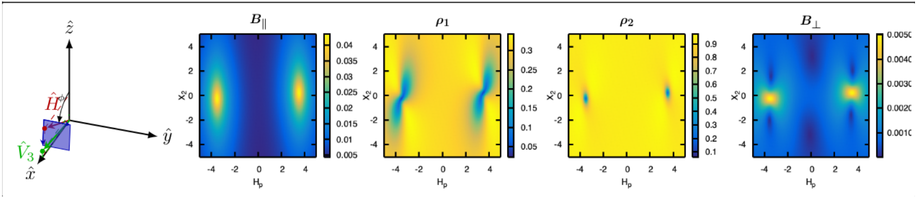
An $s + id$ model: vortex splitting



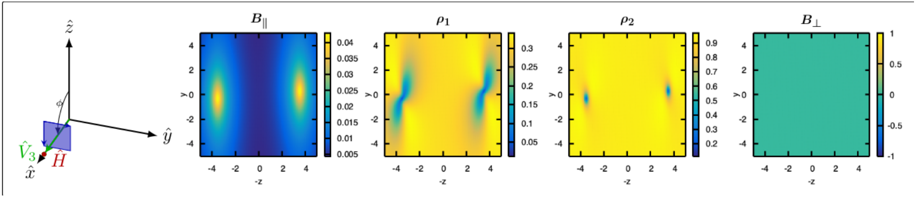
An $s + id$ model: vortex splitting



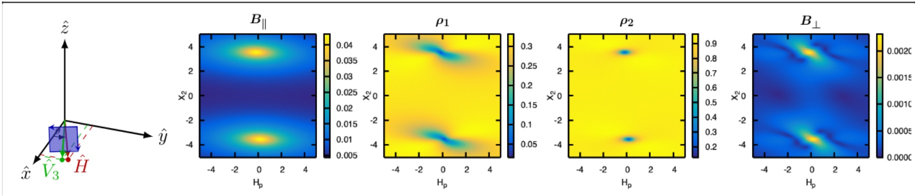
An $s + id$ model: vortex splitting



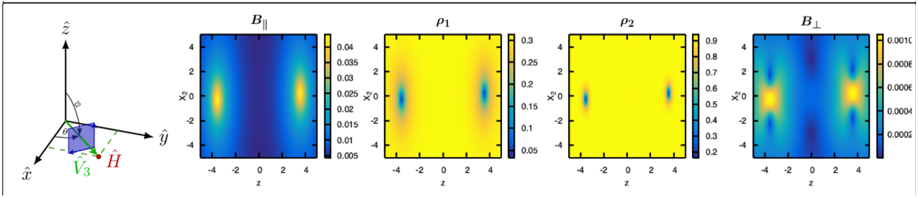
An $s + id$ model: vortex splitting



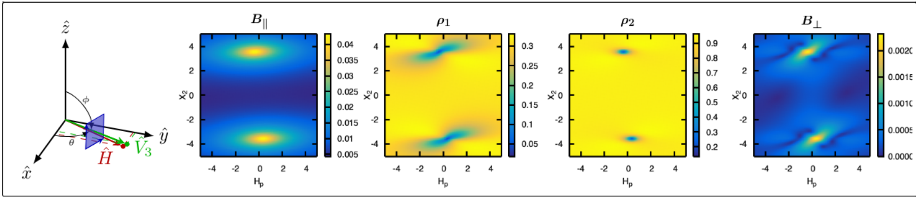
An $s + id$ model: vortex splitting



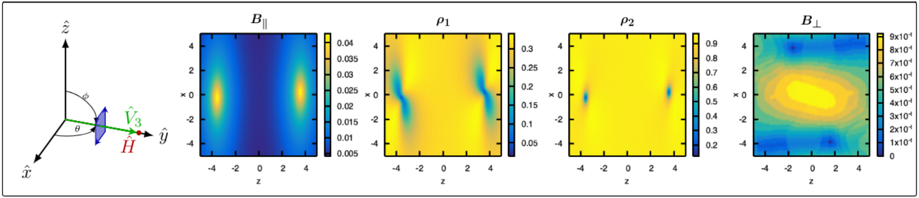
An $s + id$ model: vortex splitting



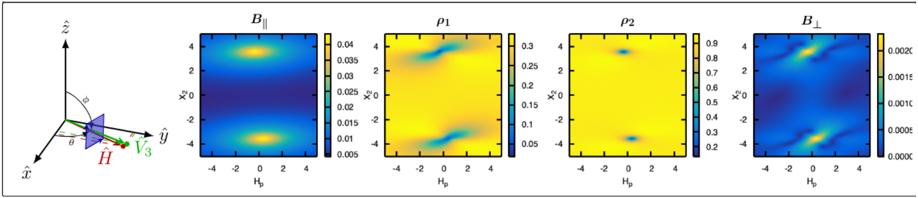
An $s + id$ model: vortex splitting



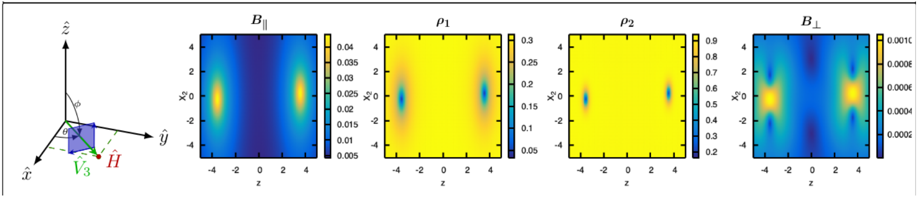
An $s + id$ model: vortex splitting



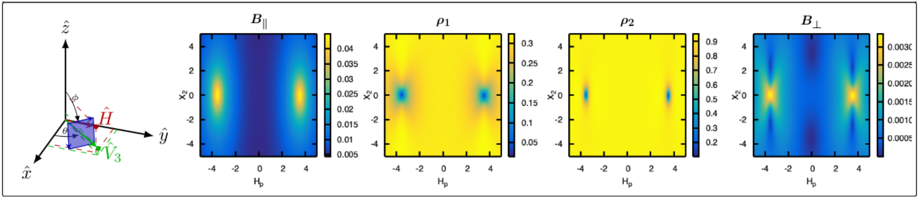
An $s + id$ model: vortex splitting



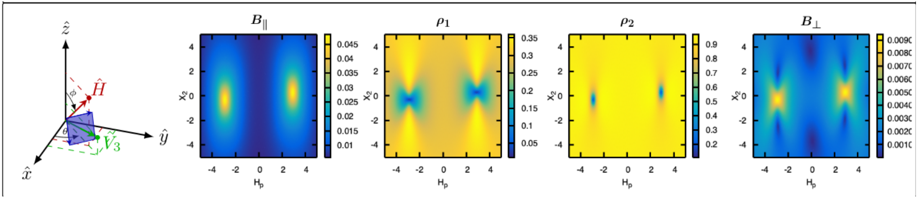
An $s + id$ model: vortex splitting



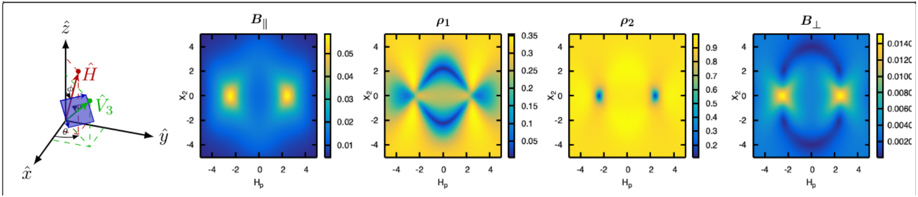
An $s + id$ model: vortex splitting



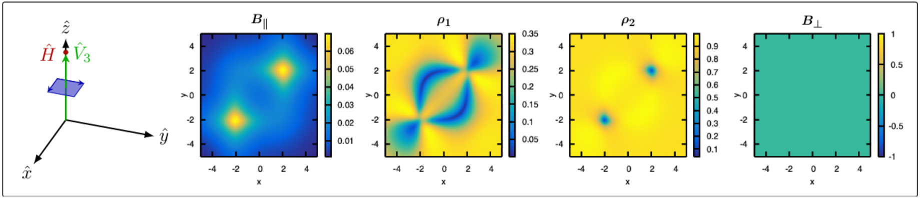
An $s + id$ model: vortex splitting



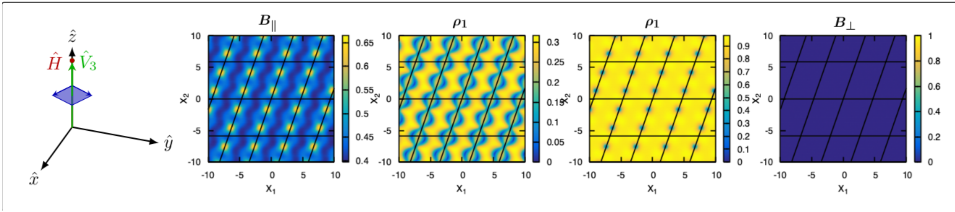
An $s + id$ model: vortex splitting



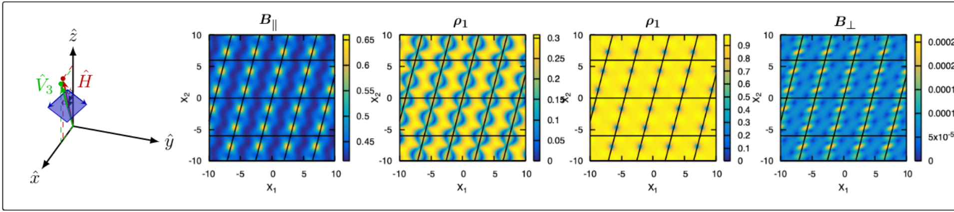
An $s + id$ model: vortex splitting



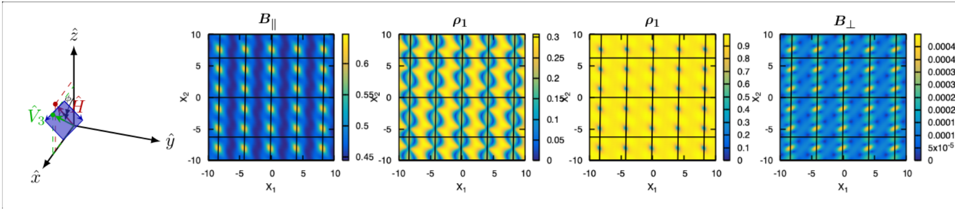
An $s + id$ model: vortex lattice at $|H| = 0.6$



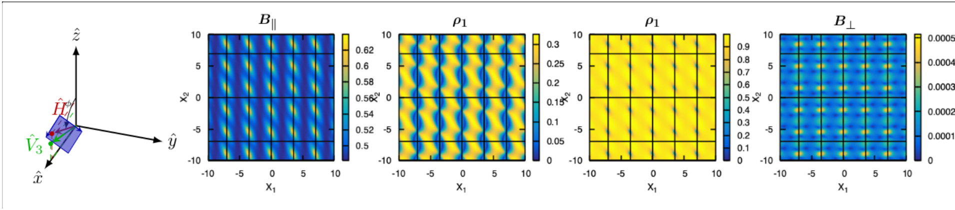
An $s + id$ model: vortex lattice at $|H| = 0.6$



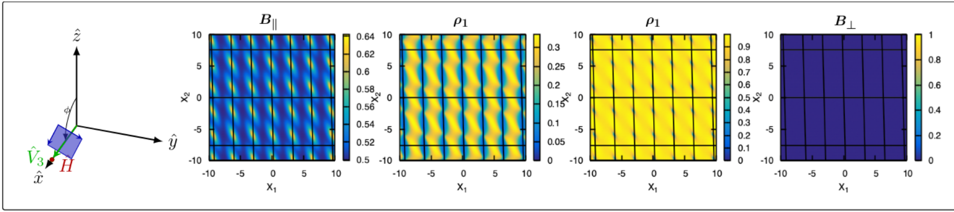
An $s + id$ model: vortex lattice at $|H| = 0.6$



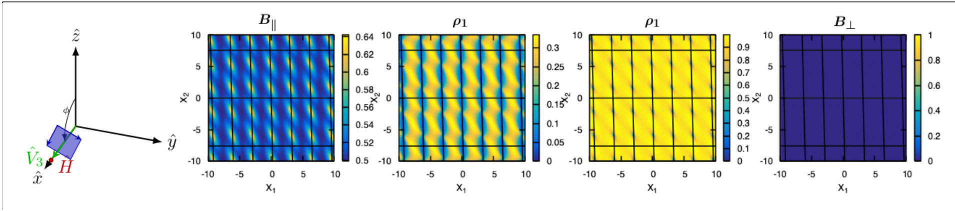
An $s + id$ model: vortex lattice at $|H| = 0.6$



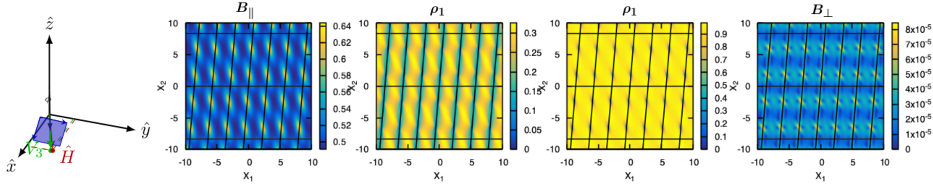
An $s + id$ model: vortex lattice at $|H| = 0.6$



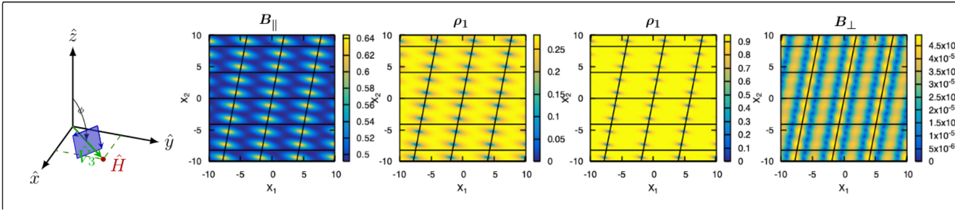
An $s + id$ model: vortex lattice at $|H| = 0.6$



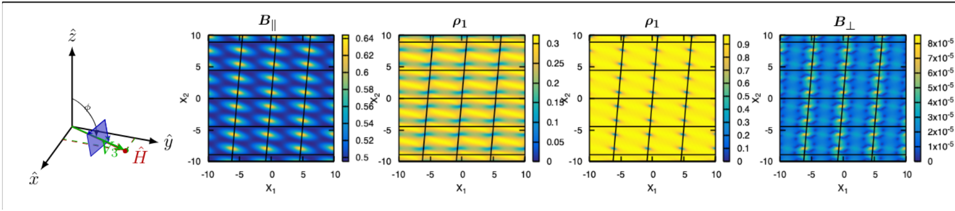
An $s + id$ model: vortex lattice at $|H| = 0.6$



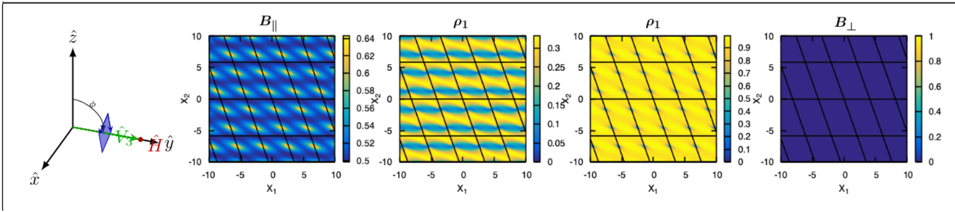
An $s + id$ model: vortex lattice at $|H| = 0.6$



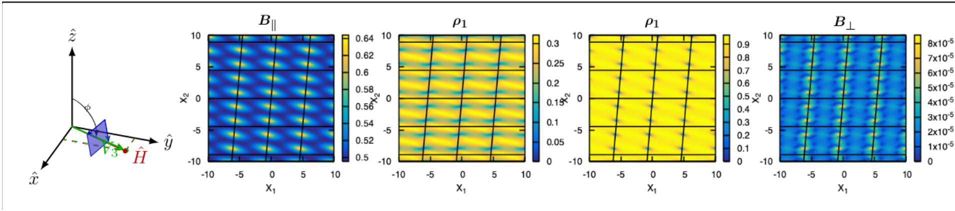
An $s + id$ model: vortex lattice at $|H| = 0.6$



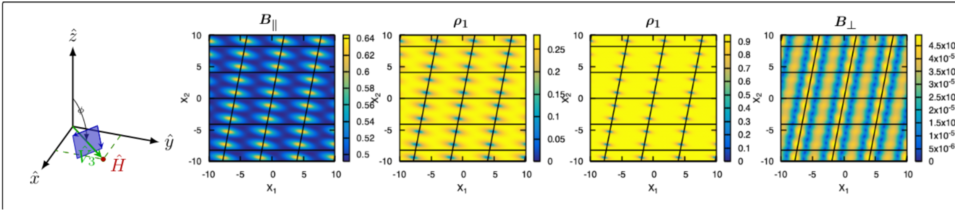
An $s + id$ model: vortex lattice at $|H| = 0.6$



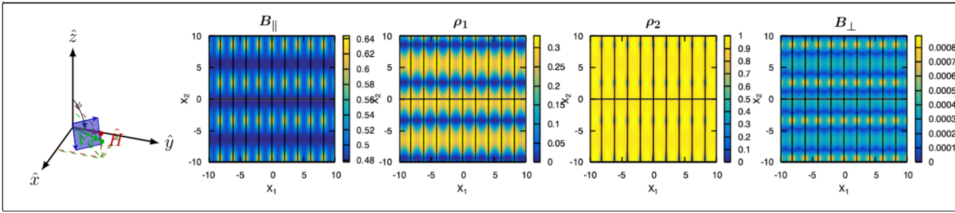
An $s + id$ model: vortex lattice at $|H| = 0.6$



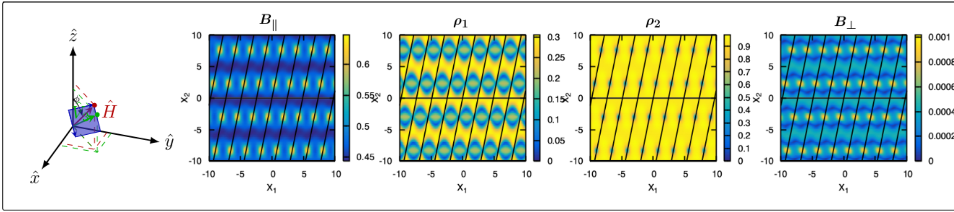
An $s + id$ model: vortex lattice at $|H| = 0.6$



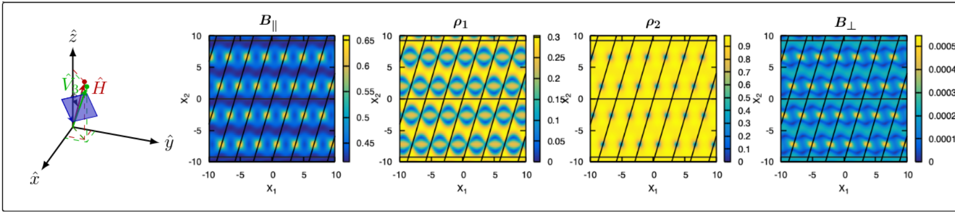
An $s + id$ model: vortex lattice at $|H| = 0.6$



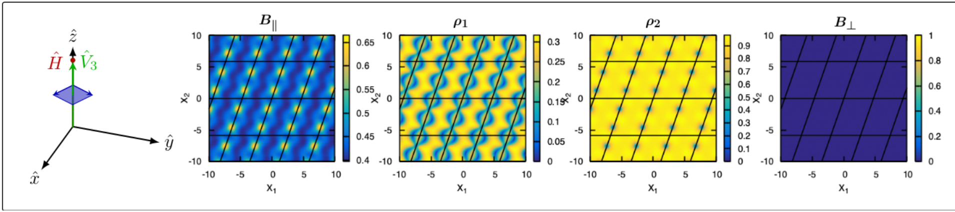
An $s + id$ model: vortex lattice at $|H| = 0.6$



An $s + id$ model: vortex lattice at $|H| = 0.6$



An $s + id$ model: vortex lattice at $|H| = 0.6$



Concluding remarks

- Vortex lattices in anisotropic superconductors are complicated
- Can't just scale the Abrikosov lattice
  - Vortex line tilting
  - Magnetic field deviation
  - Core splitting
- Need to minimize $\langle G \rangle$ over fields and period lattice $[v_1, v_2, v_3]$
- Assumption that $v_3 \parallel H$ is **not** well justified