Exotic magnetic structures in multicomponent superconductors

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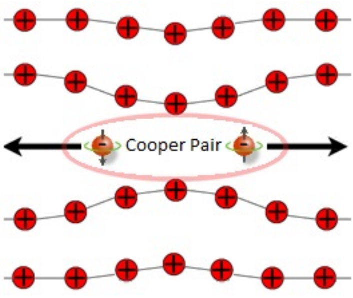
BCS theory of superconductivity



[1]

$$\text{BCS} \xrightarrow{\text{Gorkov}} \text{GL}$$

$$F = \frac{1}{2}|D_i\psi|^2 + \alpha(T)|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2}|B|^2$$

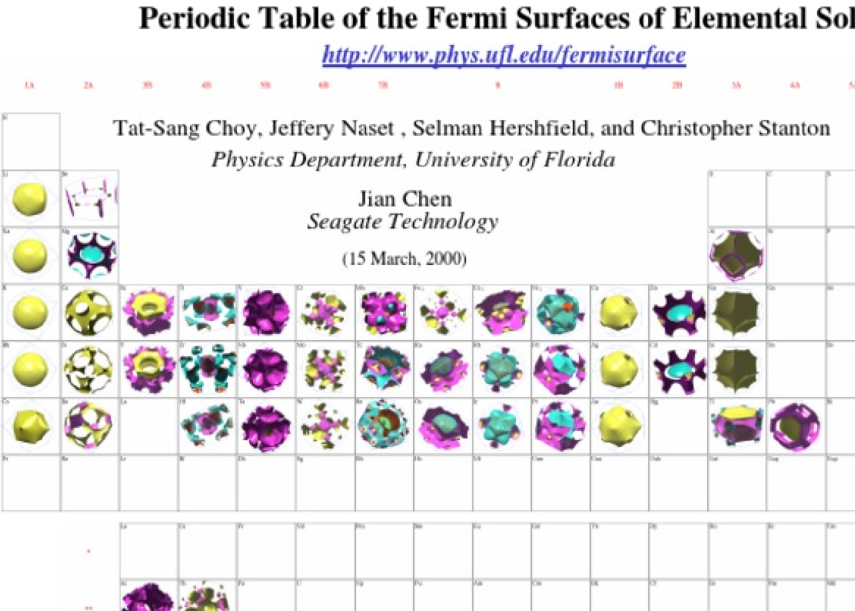
[1] Picture credit: Stuart Tessmer (Michigan State U)

Fermi surface

- Surface in momentum space: momenta of electron states at Fermi energy
- Crude approx: free Fermi gas $\Rightarrow$ sphere
- Spin singlet s-wave pairing

$$F = \frac{1}{2}|D_i\psi|^2 + \alpha(T)|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2}|B|^2$$

Fermi surface



Fermi surface

- Far from isotropic
- Multiple bands
- exotic pairing possible:
    - spin triplet p-wave
    - spin singlet d-wave
    - mix and match
- Multicomponent, anisotropic GL model

Multicomponent anisotropic GL theory

- Several condensates ψ_α, $\alpha = 1, 2, \ldots, N$. We'll stick to $N = 2$.
- Gauge potential A, $B = dA$, $D\psi = d\psi - iA\psi$

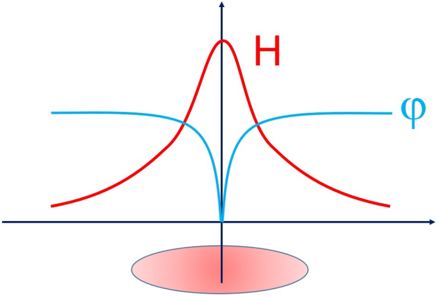
$$F = \frac{1}{2} Q_{ij}^{\alpha\beta} \overline{D_i \psi_\alpha} D_j \psi_\beta + V(\psi) + \frac{1}{2}|B|^2$$

- $Q_{ij}^{\alpha\beta} = \bar{Q}_{ji}^{\beta\alpha}$
- $V(e^{i\theta}\psi) = V(\psi)$

$$\begin{aligned} -Q_{ij}^{\alpha\beta} D_i D_j \psi_\beta + 2\frac{\partial V}{\partial \bar{\psi}_\alpha} &= 0 \\ -\partial_j(\partial_j A_i - \partial_i A_j) &= \mathrm{Im}\left(Q_{ij}^{\alpha\beta} \bar{\psi}_\alpha D_j \psi_\beta\right) \end{aligned}$$

Vortices

Conventional GL



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- Repel
- Anisotropic TCGL?

[2] Picture credit: H. Suganuma , Y. Nakagawa , K. Matsumoto (Kyoto)

Flux quantization

$$F = \int_{\mathbb{R}^2} \frac{1}{2} Q(D\psi, D\psi) + V(\psi) + \frac{1}{2}|B|^2$$

- $V \geq 0,\ V(u) = 0,\ u \neq 0$
- As $r \to \infty$, $[\psi] \to [u]$, $D\psi \to 0$

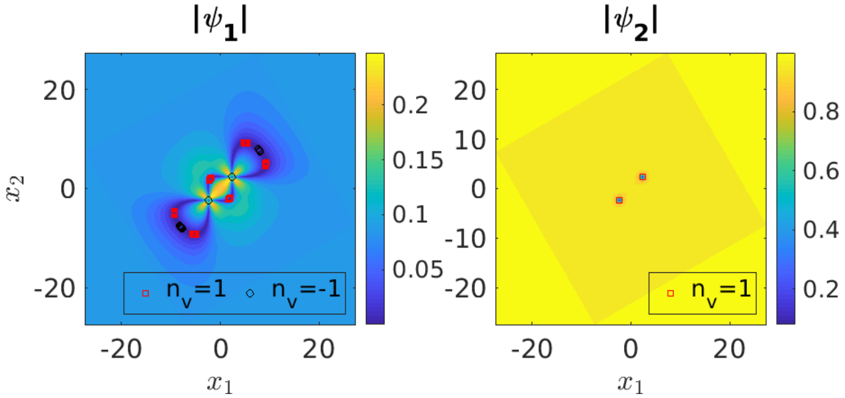
$$\psi \sim ue^{i\chi(\theta)}, \qquad A \sim d\chi$$

- Flux quantization

$$\int_{\mathbb{R}^2} B = \oint_{S^1_\infty} A = \chi(2\pi) - \chi(0) = 2\pi n$$

- Each ψ_α has n zeroes (counted with multiplicity)

It's complicated[3]



[3] Zhang et al Phys. Rev. B 101, 064501 (2020)

What's going on?

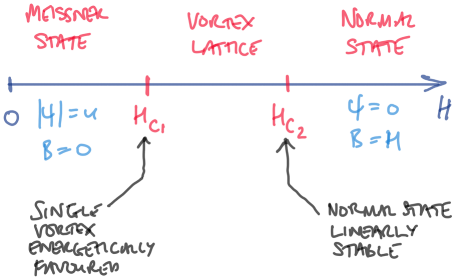
- $s + id$ model, potential tuned so that $u_1 << u_2$

$$
\begin{aligned}
F &= \cdots + Q_{ij}^{12} \overline{D_i \psi_1} D_j \psi_2 + \cdots \\
&= \cdots + (dx^2 - dy^2)(\nabla \psi_1, \nabla \psi_2) + \cdots
\end{aligned}
$$

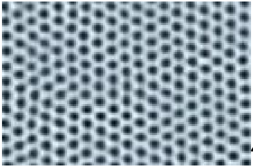
- Contributes **negatively** if $\psi_2 \sim re^{i\theta}$, $\psi_1 \sim -re^{-i\theta}$

Vortex lattices

- Create vortices by applying external magnetic field H

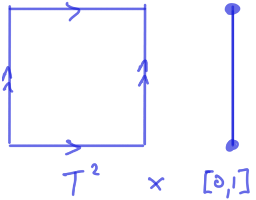


- Conventional picture: triangular vortex lattice

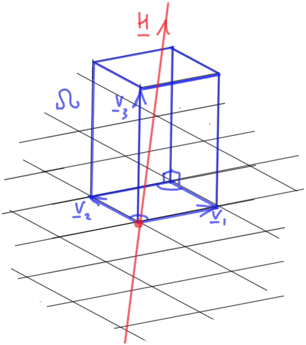
 [4]

[4] Picture credit: Somesh Chandra Ganguli (Aalto Un., Helsinki)

Optimal lattice geometry?



$$L = \overset{\longrightarrow}{\begin{pmatrix} \uparrow & \uparrow & \uparrow \\ \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 \\ \downarrow & \downarrow & \downarrow \end{pmatrix}}$$



sections ψ_α of $\mathcal{L} \to T^2 \times [0,1]$ $\longleftrightarrow$ periodic fields
connexion A on $\mathcal{L}$

Optimal lattice geometry?

Should minimize

$$
\begin{aligned}
G &= \int_\Omega \left\{ \frac{1}{2} Q(D\psi, D\psi) + V(\psi) + \frac{1}{2} |B - H|^2 \right\} \\
&= F - H \cdot \int_\Omega B + \frac{1}{2} \int_\Omega |H|^2
\end{aligned}
$$

w.r.t. ψ_α, A **and** L (**and** $n = \deg \mathcal{L}$):

$$
G = \frac{1}{2} L_{ki}^{-1} P_{ki,lj} L_{lj}^{-1} + \frac{1}{2} \mathrm{tr}\left(L \mathbb{F} L^T \right) - 2n\pi H_i L_{i3} + \int_{T^2 \times [0,1]} V(\psi),
$$

where

$$
\begin{aligned}
P_{ki,lj} &= \mathrm{Re} \int_{T^2 \times [0,1]} Q_{ij}^{\alpha\beta} \overline{D_k \psi_\alpha} D_l \psi_\beta \\
\mathbb{F}_{ij} &= \int_{T^2 \times [0,1]} F_i F_j .
\end{aligned}
$$

Numerical method

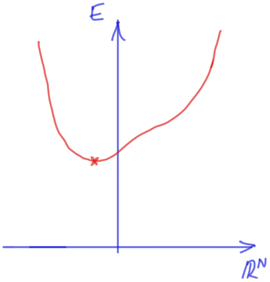
- Discretize T^2

$$G : \mathbb{R}^{7N^2} \times \mathscr{C} \to \mathbb{R}$$

where

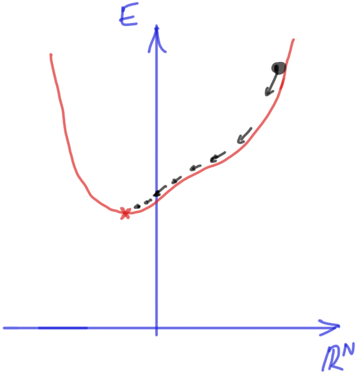
$$\mathscr{C} = \{L \in GL(3,\mathbb{R}) : \det L = 1,\ L_{i1}L_{i3} = 0,\ L_{i2}L_{i3} = 0\} \subset \mathbb{R}^9$$

- Minimize a function on a (codimension 3 submfd of) a big Euclidean space.



Gradient flow?

$$\dot{\phi}(t) = -\operatorname{grad} E(\phi(t))$$



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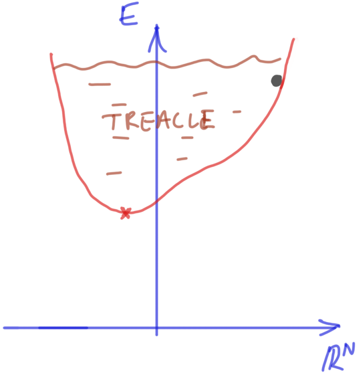
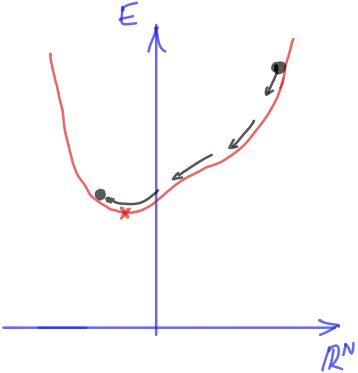


Arrested Newton flow

$$\ddot{\phi}(t) = -\operatorname{grad} E(\phi(t))$$



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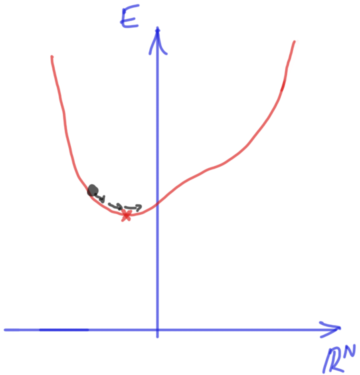


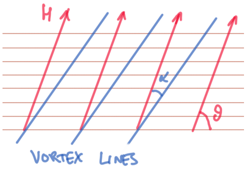
Vortex line tilting

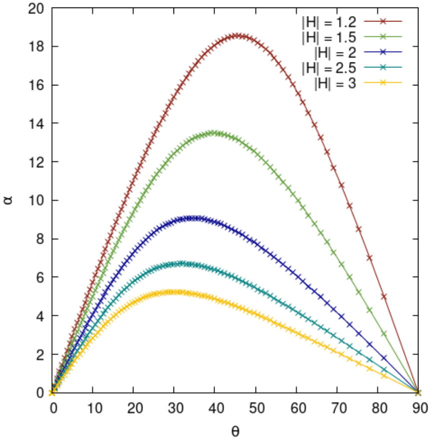
- **Single** component model:

$$Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.1 \end{pmatrix}, \qquad V = \frac{9}{4}(1 - |\psi|^2)^2$$

- Optimal lattice does **not** have $\mathbf{v}_3 \parallel H$ if H not an eigenvector of Q

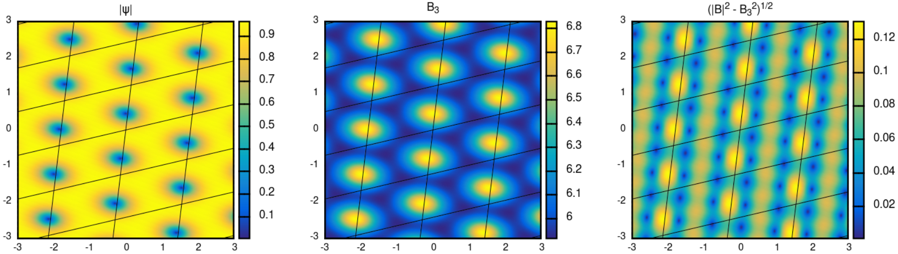
Vortex line tilting





Magnetic field twisting

$$H = 3, \qquad \theta = 40^\circ$$



Nematic superconductor[5], $H = (0, 0, |H|)$

$$Q^{11} = Q^{22} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad Q^{12} = (Q^{21})^\dagger = \beta \begin{pmatrix} 1 & i \\ i & -1 \end{pmatrix}$$

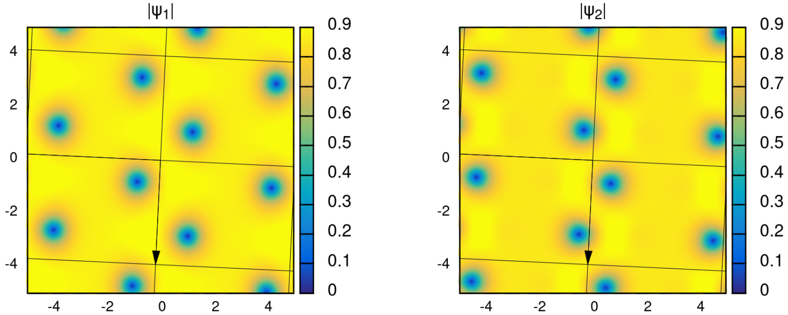
$$V = V_0 \left(-|\psi|^2 + \frac{1}{2}|\psi|^4 + (\gamma - 1)|\psi_1|^2|\psi_2|^2 \right)$$

- $\gamma \in (-1, 1)$: torus of vacua $\psi = (u^{i\alpha}, u^{i\beta})$
- $n = 2$ lattices have least $G/|\Omega|$
- E.g. $V_0 = 9/2$, $\beta = 1/3$, $\gamma = 1/3$

[5] Model derived by Venderbos, Kozii, Fu

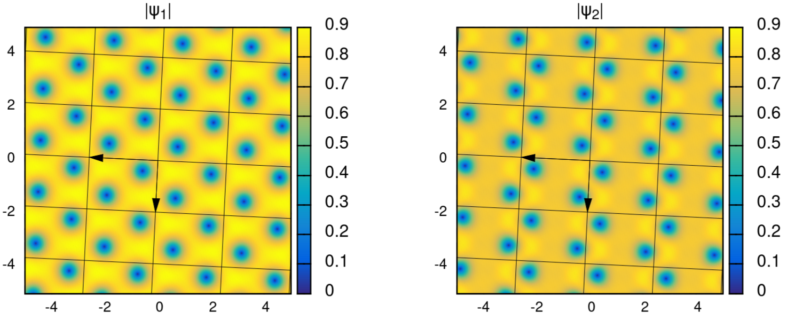
Nematic vortex lattices: condensates

$$H = 1.313$$



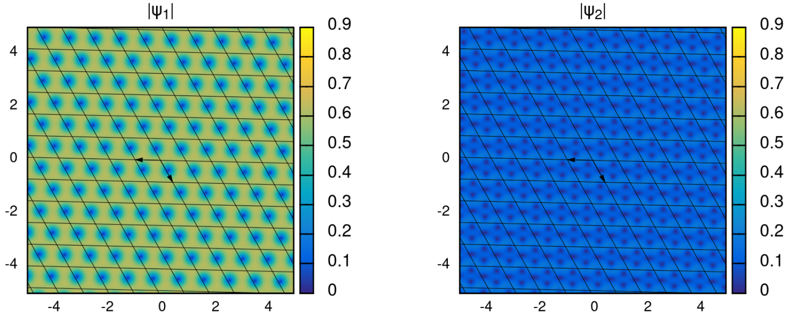
Nematic vortex lattices: condensates

$$H = 2.93243$$



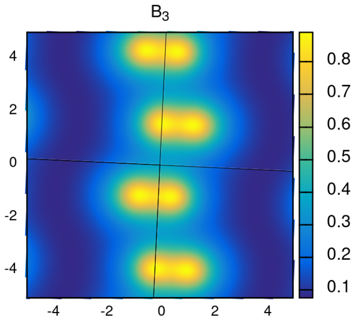
Nematic vortex lattices: condensates

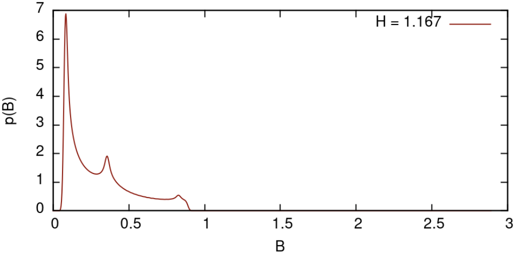
$$H = 8.0$$



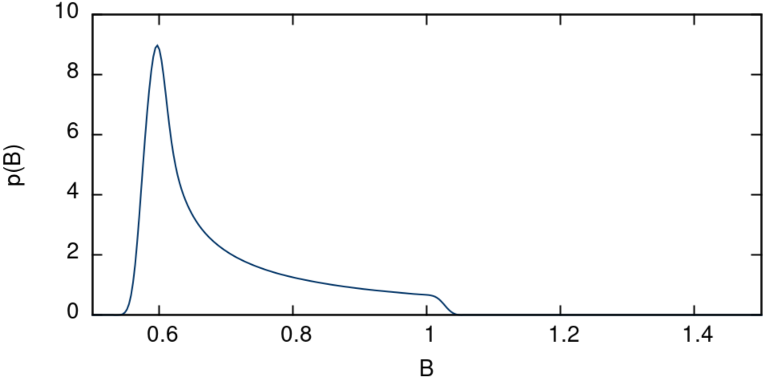
Nematic vortex lattices: B

$$H = 1.167$$



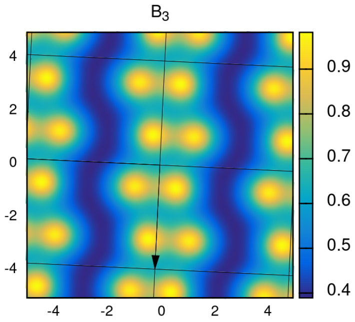


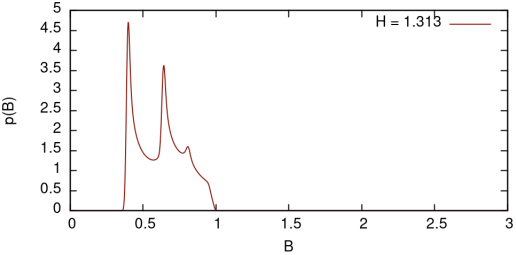
Abrikosov lattice $H = 1$



Nematic vortex lattices: B

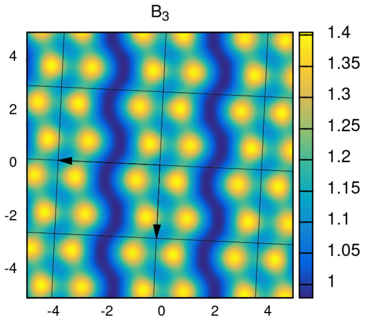
$$H = 1.313$$

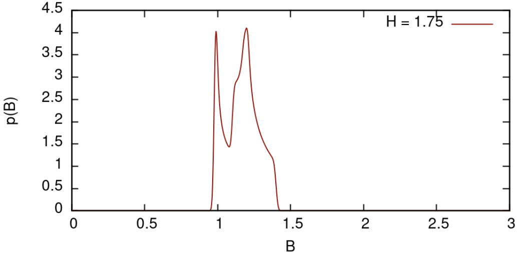




Nematic vortex lattices: B

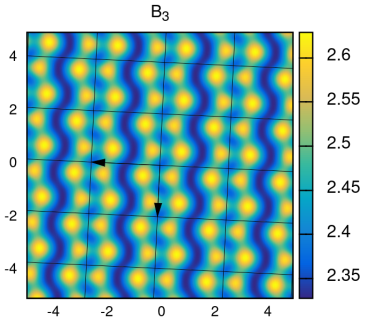
$$H = 1.750$$

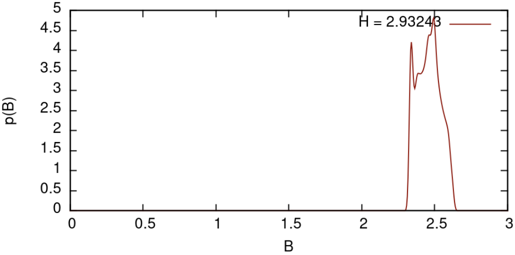




Nematic vortex lattices: B

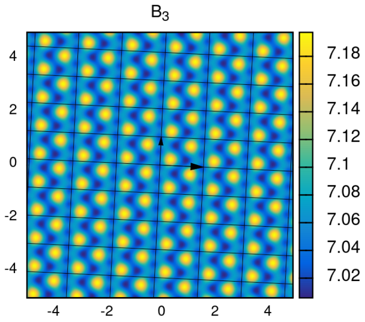
$$H = 2.93243$$

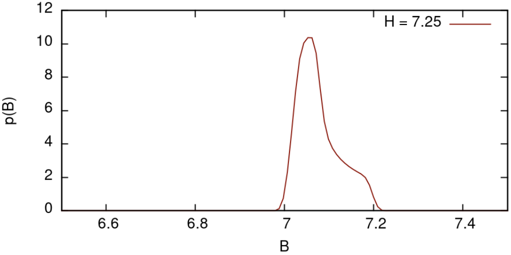




Nematic vortex lattices: B

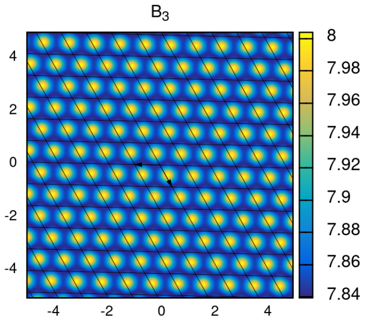
$$H = 7.25$$

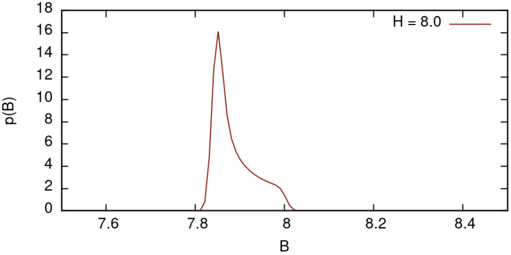




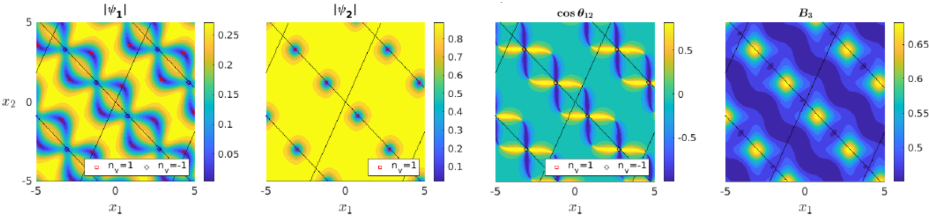
Nematic vortex lattices: B

$$H = 8.0$$



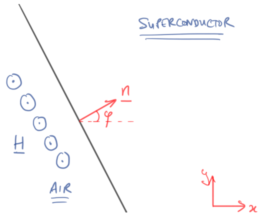


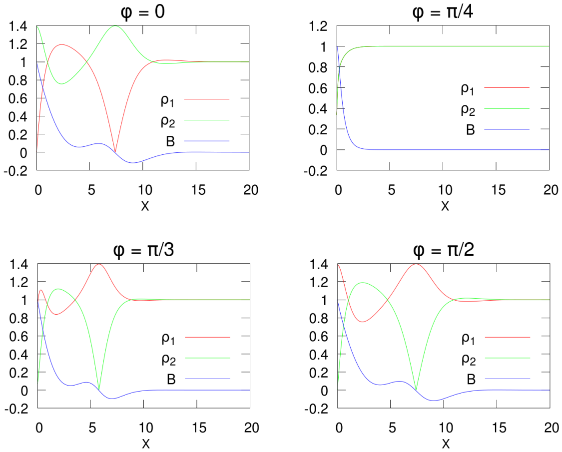
Freaky $s + id$ vortex lattice

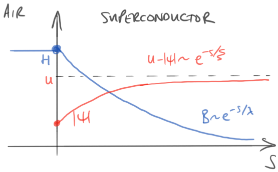


Meissner state in a $p + ip$ model[6]

$$Q^{11} = \begin{pmatrix} 3+\nu & 0 \\ 0 & 1-\nu \end{pmatrix}, \; Q^{22} = \begin{pmatrix} 1-\nu & 0 \\ 0 & 3+\nu \end{pmatrix}, \; Q^{12} = \begin{pmatrix} 0 & 1-\nu \\ 1-\nu & 0 \end{pmatrix}$$







[6] Model proposed by Bouhon and Sigrist

Concluding remarks

Phenomenology of anisotropic TCGL is very rich and poorly understood.

- Full coupling, complex length scales, oscillatory spatial decay
- BTRS, domain walls, orientation dependence , B twisting
- Very exotic vortices/skyrmions
- Nonmonotonic vortex interactions
- Complicated lattices
- Vortex line tilting
- Anisotropic H_{c1}, H_{c2}