Long range vortex interactions and type 1.5 superconductivity

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Single component GL theory

- $\psi : \mathbb{R}^2 \to \mathbb{C}$, gauge field A, magnetic field $B = \mathrm{d}A$

$$E = \int_{\mathbb{R}^2} \frac{1}{2}|\mathrm{d}_A\psi|^2 + \frac{1}{2}|B|^2 + \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4$$

  where $\mathrm{d}_A\psi = \mathrm{d}\psi - iA\psi$
- If temperature $T < T_c$ then $\alpha < 0$
  - Normal state $\psi = 0$ unstable
  - Meissner state $|\psi| = \sqrt{\frac{|\alpha|}{\beta}}$
- Rescale: one remaining parameter $\lambda > 0$

$$E = \int_{\mathbb{R}^2} \frac{1}{2}|\mathrm{d}_A\psi|^2 + \frac{1}{2}|B|^2 + \frac{\lambda}{8}(1 - |\psi|^2)^2$$

Vortices

$$E = \int_{\mathbb{R}^2} \frac{1}{2}|\mathrm{d}_A\psi|^2 + \frac{1}{2}|B|^2 + \frac{\lambda}{8}(1 - |\psi|^2)^2$$

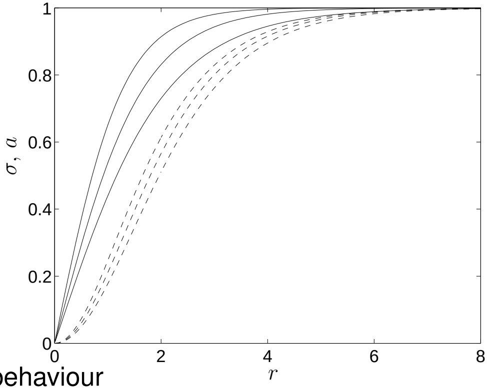
- Finite energy:
  - $\psi : S^1_\infty \to S^1 \subset \mathbb{C}$, winding n
  - $\mathrm{d}_A\psi(\infty) = 0 \quad \Rightarrow \quad A_\infty = \mathrm{d}(arg(\psi_\infty))$
  - Stokes's theorem: $\Phi = \int_{\mathbb{R}^2} B = \int_{S^1_\infty} A = 2\pi n$
- Field equations

$$\begin{aligned} -*\mathrm{d}_A*\mathrm{d}_A\psi - \frac{\lambda}{2}(1-|\psi|^2)\psi &= 0 \\ -*\mathrm{d}*B &= Im(\overline{\psi}\mathrm{d}_A\psi) \end{aligned}$$

- Vortex ansatz $\psi = \sigma(r)e^{in\theta}, \qquad A = a(r)\mathrm{d}\theta$

  Reduces to ODEs

Vortices



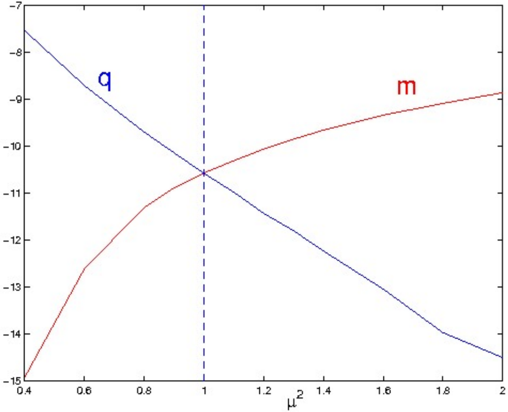
- Large r behaviour

$$\psi = \left(1 + \frac{q}{2\pi} K_0(\sqrt{\lambda} r) + \cdots \right) e^{i\theta}$$

$$A = \left(1 + \frac{m}{2\pi} r K_1(r) + \cdots \right) \mathrm{d}\theta$$

- Constants $q(\lambda), m(\lambda)$ determined numerically

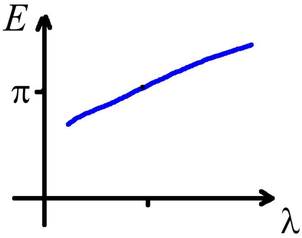
Single component GL theory

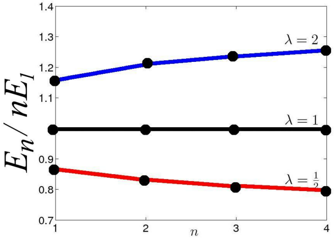


- $q(1) \equiv m(1)$

Type I / Type II dichotomy

- $E_{n=1}(\lambda)$ monotonically increasing, $E_{n=1}(1) = \pi$

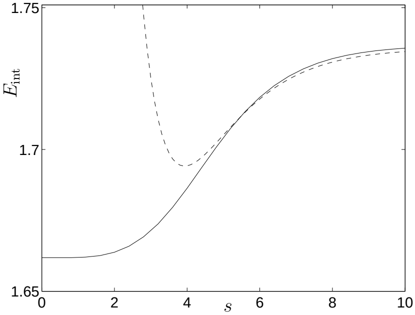


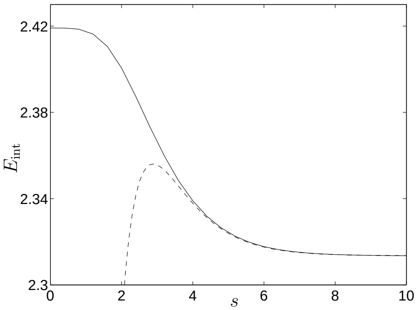


- Stability for $n > 1$ depends on λ:
  - $\lambda > 1$: $E_2 > 2E_1$ so coincident vortices release energy by separating
  - $\lambda < 1$: $E_2 < 2E_1$ well-separated vortices release energy by coalescing
- Dichotomy: $\lambda < 1$ Type I, $\lambda > 1$ Type II

Type I / Type II dichotomy

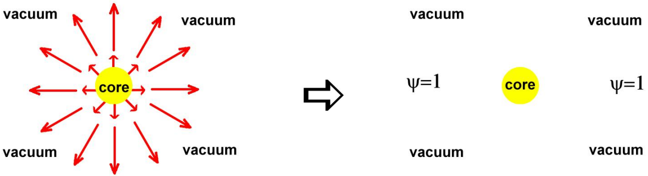
- Two vortex interaction energy





Point vortex model

- Gauge choice: ψ real (unique vacuum $\psi = 1$)
- Unwind static vortex with singular gauge transformation



$$\hat{\psi} = \psi - 1 = \frac{q}{2\pi} K_0(\sqrt{\lambda} r) + \cdots, \qquad A = \frac{m}{2\pi} r K_1(r) \mathrm{d}\theta + \cdots$$

Point vortex model

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- Coincides asymptotically with solution of **linearization** of model about the vacuum $\widehat{\psi} = 0$

$$\mathcal{L} = \frac{1}{2}\partial_\mu \widehat{\psi} \partial^\mu \widehat{\psi} - \frac{\lambda}{2}\widehat{\psi}^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}A_\mu A^\mu + \kappa\widehat{\psi} - j_\mu A^\mu$$

in the presence of point sources

$$\begin{aligned} \kappa &= q\delta(x) && \text{scalar monopole} \\ j^\mu &= m(0, \partial_y, -\partial_x)\delta(x) && \text{magnetic dipole} \end{aligned}$$

Point vortex model

- Intervortex forces at long range should approach those between two such ”point vortices” interacting via the linear theory

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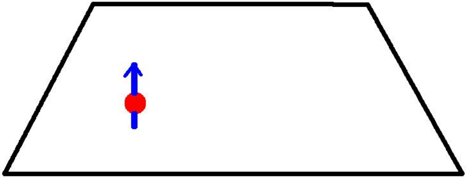

Point vortex model

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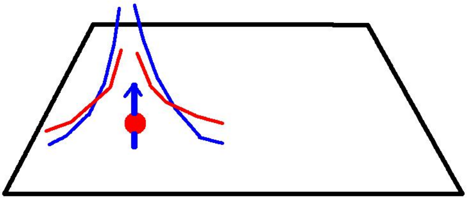

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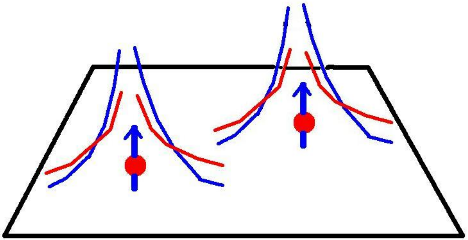


Point vortex model

- Interaction between sources κ_1, κ_2 for a real scalar field $\widehat{\psi}$ of mass $\sqrt{\lambda}$

$$L_{int} = \int_{\mathbb{R}^2} \kappa_1 \widehat{\psi}_2 \qquad \text{where} \qquad \Delta\widehat{\psi}_2 + \lambda\widehat{\psi}_2 = \kappa_2$$

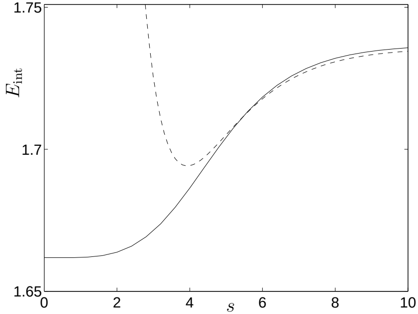
- Two static monopoles $\kappa_1(x) = q\delta(x-y)$, $\kappa_2(x) = q\delta(x-z)$

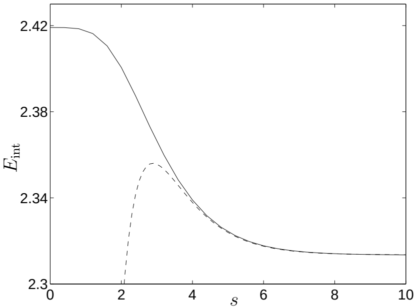
$$L_{int} = \int_{\mathbb{R}^2} q\delta(x-y)\frac{q}{2\pi}K_0(\sqrt{\lambda}|x-z|)d^2x = \frac{q^2}{2\pi}K_0(\sqrt{\lambda}|y-z|)$$

- Interaction of vector dipoles similar
- Total interaction energy ($R = |y-z|$)

$$E_{int}(R) = \frac{1}{2\pi}\left[m^2 K_0(R) - q^2 K_0(\sqrt{\lambda}R)\right]$$

Point vortex model

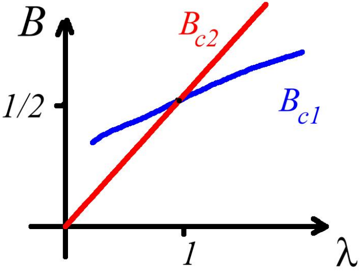




- Reproduces type I/II dichotomy

Critical applied fields

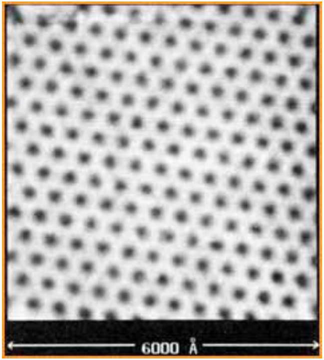
- B_{c1} = field required to produce a vortex $= \frac{E_{vortex}}{2\pi}$
- B_{c2} = min field for which trivial soln ($\psi = 0$, $B = const$)is stable $= \frac{\lambda}{2}$



- Type I: field required to produce vortex is **larger** than field at which normal state is stable: never produce vortices
- Type II: field window $[B_{c1}, B_{c2}]$ where vortex creation is possible

Critical applied fields

- Vortices, if they occur at all, always repel. Abrikosov vortex lattice



NbSe2, 1T, 1.8K (H. F. Hess et al., Phys. Rev. Lett. 62, 214 (1989))

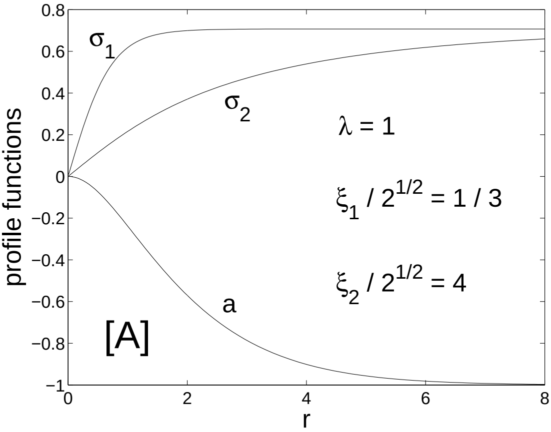
Two-component GL theory

- Some superconductors have more than one electron-pair condensate
- Toy model

$$E = \int_{\mathbb{R}^2} \frac{1}{2}|B|^2 + \sum_{i=1,2} \frac{1}{2}|\mathrm{d}_A\psi_i|^2 - \alpha_i|\psi_i|^2 + \frac{1}{2}\beta_i|\psi_i|^4$$

- Meissner state $|\psi_i| = u_i = \sqrt{\dfrac{\alpha_i}{\beta_i}}$
- Linearization about Meissner state:
  - real scalar bosons $\widehat{\psi}_i = |\psi_i| - u_i$, masses $\mu_i = 2\sqrt{\alpha_i}$
  - real scalar boson $\Delta = arg(\psi_1) - arg(\psi_2)$, massless
  - vector boson of mass $\mu_A = \sqrt{u_1^2 + u_2^2}$
- Vortices as before ($n_1 = n_2$)

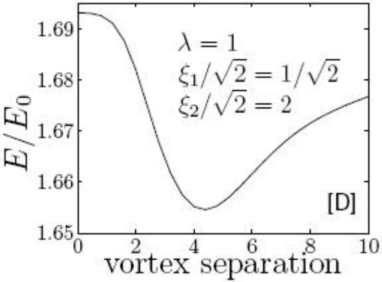
Two-component GL theory



Vortex interactions

- "Point vortex" carries
  - two scalar monopole charges q_i, inducing scalar fields $\widehat{\psi}_i$ of mass μ_i
  - no source for Δ
  - magnetic dipole moment m, inducing vector field A of mass $\mu_A = \sqrt{u_1^2 + u_2^2}$
- Interaction energy: sum of three terms
- Long range attraction if $\min\{\mu_1, \mu_2\} < \mu_A$
- Large regions of parameter space for which this coincides with short range **repulsion**. Naive expectation: this happens when $\mu_1 < \mu_A < \mu_2$

Vortex interactions



- Critical fields

$$B_{c1} = \frac{E_{vortex}}{2\pi}, \qquad B_{c2} = \max\{2\alpha_1, 2\alpha_2\}$$

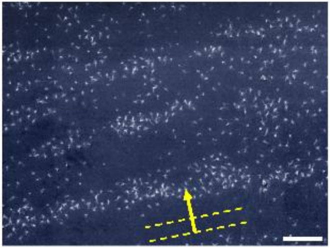
Nonmonotic regime intersects $B_{c1} < B_{c2}$ regime

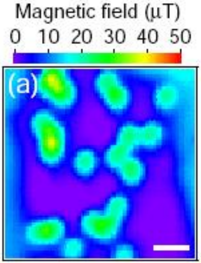
Type 1.5 superconductivity?

- Imagine we have a superconductor described by TCGL and we turn up an applied magnetic field H.
- When H reaches B_{c1} it becomes energetically favourable for magnetic flux to penetrate in a vortex (like type II)
- Increasing H, until we reach B_{c2}, more and more vortices penetrate (like type II)...
- ...**but** it's energetically favourable for the vortices to clump together at a fixed separation, rather than form a regular lattice of increasing density (not like type II)
- Predict clumps of flux penetration in a sea of Meissner state (like type I)...
- ...**but** within each clump, flux will penetrate in a vortex lattice of fixed unit cell size (not like type I)
- We called it "semi-Meissner state"...

Type 1.5 superconductivity?

- ...Moshchalkov et al found similar structure in MgB_2





$H = 5$ Oe, Bitter decoration $H = 10\mu T$, SQUID microscopy

- They called it “type 1.5 superconductivity”
- Not universally accepted (existence, or explanation).

Interband couplings

- Main criticism of our analysis: having no direct coupling between condensates is very unrealistic (only interaction is via em field). In real superconductors, have
  - direct coupling through Josephson effect

$$V_{Jos} = -\frac{\eta_1}{2}(\overline{\psi_1}\psi_2 + \psi_1\overline{\psi_2})$$

  - gradient-gradient coupling (except in ultra clean samples) due to electron scattering off impurities

$$\nu Re(d_A\psi_1, \overline{d_A\psi_2})$$

  - Also, if we're including terms up to order 4, why don't we include

$$V_{Quartic} = \eta_2|\psi_1|^2|\psi_2|^2?$$

- Once condensates are coupled, expect this to equalize their decay rates as $r \longrightarrow \infty$. Maybe this eliminates the type 1.5 regime altogether?

Interband couplings

- Riposte: direct coupling terms are forbidden in many interesting systems (e.g. liquid metallic hydrogen), so our original analysis is still relevant to such systems.
- Better riposte: the length scales of interest are inverse masses of the (now mixed) normal modes **not** the condensates themselves.
  - Let $V = V_1(\psi_1) + V_2(\psi_2) + V_{Jos}(\psi_1, \psi_2) + V_{Quartic}(\psi_1, \psi_2)$
    Then μ_1^2, μ_2^2 are eigenvalues of

$$\mathcal{H}_{ij} = \left.\frac{\partial^2 V}{\partial|\psi|_i \partial|\psi|_j}\right|_{|\psi_i|=u_i, \Delta=0}$$

  - Linearize theory about vacuum (u_1, u_2)

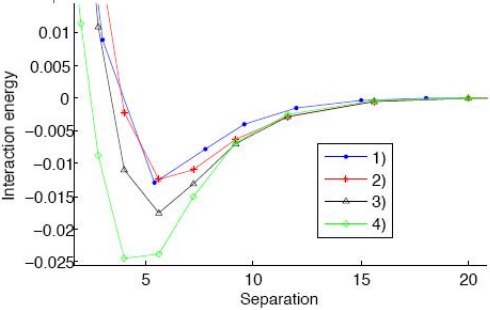
$$\begin{pmatrix} |\psi_1| \\ |\psi_2| \end{pmatrix} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} + \chi_1 e_1 + \chi_2 e_2, \qquad \mathcal{H} e_i = \mu_i^2 e_i$$

  - Get decoupled theory of real scalar fields χ_i (mixed normal modes), masses μ_i, and vector boson A, mass $\mu_A = \sqrt{u_1^2 + u_2^2}$.
  - Can still have splitting $\mu_1 < \mu_A < \mu_2$

Interband couplings

- Even better riposte: large scale numerical simulations of the model including all these extra terms show that there are big regions of parameter space where vortex interaction is non-monotonic [Babaev, Carlstrom, JMS].

Curve	α_1	β_1	α_2	β_2	η_1	η_2	e	$\langle\lvert\psi_1\rvert^2\rangle$	$\langle\lvert\psi_2\rvert^2\rangle$
1	-1	1	-0.0069	0.0278	0.0222	0	0.7	1.0094	0.723
2	-1	1	-0.0069	0.0278	0	-0.0077	0.7	1.004	0.526
3	-1	1	-0.0069	0.0278	0	-0.013	0.7	1.0094	0.72
4	-1	1	-0.0069	0.0278	0	-0.0232	0.7	1.026	1.1



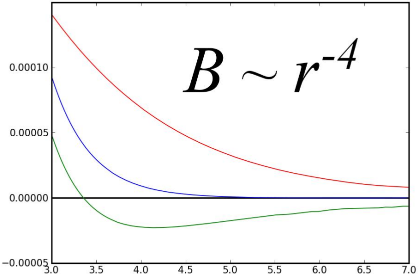
Interband couplings

- Doesn't answer question of whether MgB_2 supports type 1.5 superconductivity (have no idea what the interband coupling parameters are). But it does show that type 1.5 superconductivity is possible in principle.

Concluding remarks

- Multicomponent GL theory exhibits lots more interesting phenomena
- If V independent of Δ, fractional flux vortices

$$\psi_i = f_i(r)e^{in_i\theta} \qquad \Rightarrow \Phi = \frac{n_1 u_1^2 + n_2 u_2^2}{u_1^2 + u_2^2}$$



[Babaev, Jäykkä,JMS]

Concluding remarks

- Large gradient-gradient coupling encourages phase anti-locking where gradients are large. With opposed Josephson term, vortices get "twisted": zeros of ψ_i slip apart. "Skyrmion"

$$[\psi_1, \psi_2] : \mathbb{R}^2 \to \mathbb{C}P^1 \cong S^2$$

- **Three** component model with quadratic Josephson terms can exhibit **phase frustration** and $\mathbb{C}P^2$ "Skyrmions"

$$\varphi = [\psi_1, \psi_2, \psi_3] : \mathbb{R}^2 \to \mathbb{C}P^2$$

Flux quantization $\Phi/2\pi = [\varphi^*\omega] \in H^2(\mathbb{R}^2 \cup \{\infty\}) = \mathbb{Z}$

- Detailed energetics of vortex "molecules" unexplored (cf baby Skyrmions)
- Effect of quartic phase coupling terms unexplored

$$V_{Jos4} = \eta_3(\psi_1^2\overline{\psi}_2^2 + \overline{\psi}_1^2\psi_2^2)$$

- These solitons **actually exist in nature** and present many interesting open problems!