Complex length scales in anisotropic multicomponent superconductors

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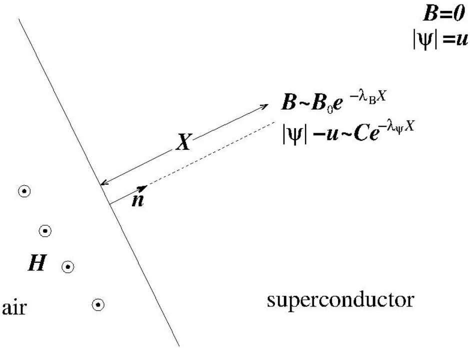
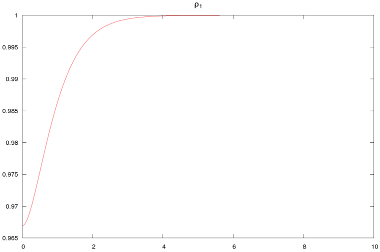
Conventional GL theory (isotropic, single component)

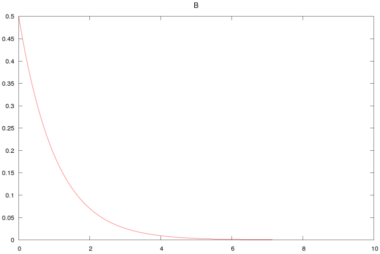


- Two length scales λ_B^{-1}, λ_ψ^{-1}
- $\lambda_B < \lambda_\psi$ type II, $\lambda_B > \lambda\psi$ type I.

Conventional GL theory (isotropic, single component)

$$F = \frac{1}{2}|D_i\psi|^2 + \frac{1}{2}|B|^2 - \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4$$





- $D_i\psi := \partial_i\psi + iA_i\psi$
- Minimize $G = \int(F - HB)$ with natural b.c.

Anisotropic multicomponent GL theory

$$F = \frac{1}{2} Q_{ij}^{\alpha\beta} (D_i \psi_\alpha)^* D_j \psi_\beta + \frac{1}{2}|B|^2 + V(\psi_1, \ldots, \psi_N)$$

- Much more diverse/interesting behaviour. $N = 2$ suffices
- BTRS particularly interesting

$$\begin{aligned} V(\psi_1, \psi_2) &= V_0(|\psi_1|, |\psi_2|) + \frac{1}{2}\eta({\psi_1^*}^2 \psi_2^2 + \psi_1^2 {\psi_2^*}^2) \\ &= V_0(\rho_1, \rho_2) + \eta \rho_1^2 \rho_2^2 \cos 2(\theta_1 - \theta_2) \end{aligned}$$

Minimized when $\theta_1 - \theta_2 = \pm\pi/2$

- Such models proposed for Sr2RuO4 (Sigrist et al)

Extracting the length scales

- Euler-Lagrange eqns for F:

$$\begin{aligned}
-Q_{ij}^{\alpha\beta} D_i D_j \psi_\beta + 2\frac{\partial V}{\partial \psi_\alpha^*} &= 0 \\
(\mathrm{curl}\, B)_i &= -\mathrm{Im}\, Q_{ij}^{\alpha\beta} \psi_\alpha^* D_j \psi_\beta
\end{aligned}$$

Extracting the length scales

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$$
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\end{aligned}
$$

- Fix direction $n = (n_1, n_2)$, impose translation symmetry

$$
\psi_1(X), \quad \psi_2(X), \quad A_i = a(X) n_i^\perp, \qquad X := n_i x_i
$$

Coupled system of nonlinear 2nd order **ODEs** for ψ_1, ψ_2, a

Extracting the length scales

- Euler-Lagrange eqns for F:

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- Fix direction $n = (n_1, n_2)$, impose translation symmetry

$$\psi_1(X), \quad \psi_2(X), \quad A_i = a(X) n_i^\perp, \qquad X := n_i x_i$$

Coupled system of nonlinear 2nd order **ODEs** for ψ_1, ψ_2, a

- Linearize about vacuum:

$$|\psi_1| = u_1 + \varepsilon_1, \quad |\psi_2| = u_2 + \varepsilon_2, \quad \theta_1 - \theta_2 = \frac{\pi}{2} + \Theta, \quad a$$

- Coupled system of **linear** 2nd order ODEs for $\varepsilon_1, \varepsilon_2, \Theta, a$

Extracting the length scales

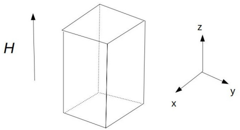
$$\mathscr{A}(n)\frac{d^2\xi}{dX^2} + \mathscr{B}(n)\frac{d\xi}{dX} + \mathscr{C}(n) = 0, \qquad \xi = (\varepsilon_1, \varepsilon_2, \Theta, a)$$

- $\mathscr{A}^T = \mathscr{A}$, $\mathscr{B}^T = -\mathscr{B}$, $\mathscr{C}^T = \mathscr{C}$, n dependent
- General solution: superposition of $\xi = ve^{-\lambda X}$ where

$$\det(\mathscr{A}\lambda^2 - \mathscr{B}\lambda + \mathscr{C}) = 0$$

- No reason why eigenvalues must be **real**!
- Dominant eigenvalue $\lambda_* = \lambda_1 + i\lambda_2$ has smallest $\lambda_1 > 0$
- Large X behaviour: $\xi \sim v_* e^{-\lambda_1 X} \cos \lambda_2 X$

Example: the Bouhon-Sigrist model of Sr2RuO4

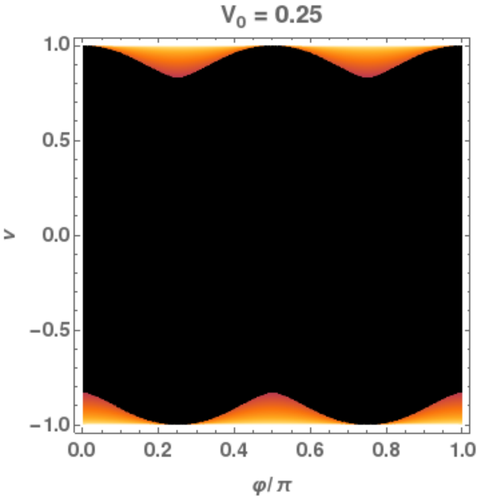


$$Q^{11}=\begin{pmatrix} 3+\nu & 0 \\ 0 & 1-\nu \end{pmatrix},\quad Q^{22}=\begin{pmatrix} 1-\nu & 0 \\ 0 & 3+\nu \end{pmatrix},\quad Q^{12}=Q^{21}=\begin{pmatrix} 0 & 1-\nu \\ 1-\nu & 0 \end{pmatrix},$$

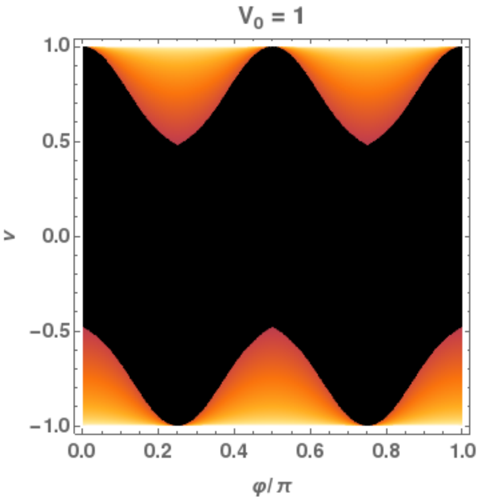
$$V=V_0\left\{1-\rho_1^2-\rho_2^2+\tfrac{3+\nu}{8}(\rho_1^2+\rho_2^2)^2-\tfrac{1+3\nu}{4}\rho_1^2\rho_2^2+\tfrac{1-\nu}{4}\rho_1^2\rho_2^2\cos 2(\theta_1-\theta_2)\right\}$$

- $-1<\nu<1$ anisotropy parameter
- V_0 potential energy scale

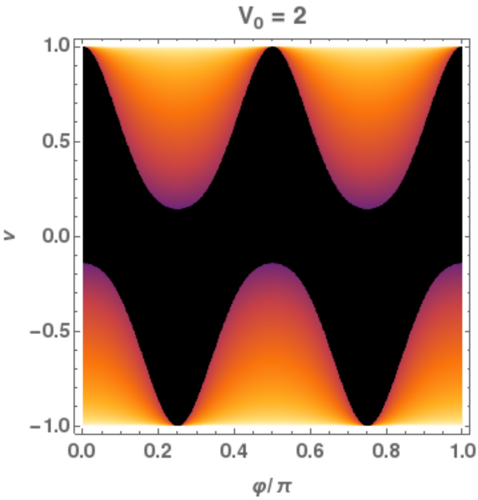
Dominant eigenvalue



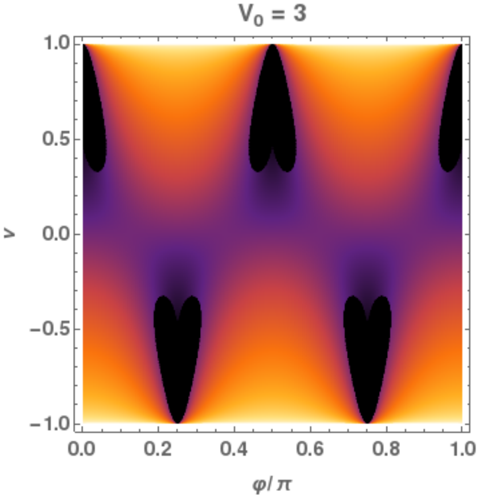
Dominant eigenvalue



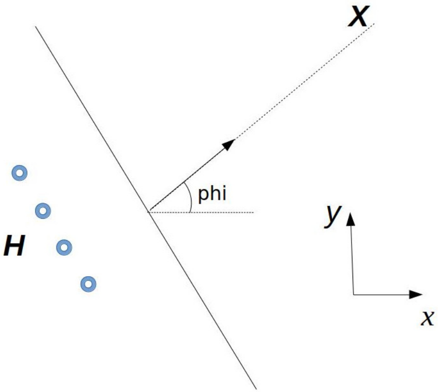
Dominant eigenvalue



Dominant eigenvalue



Numerical test: Meissner state



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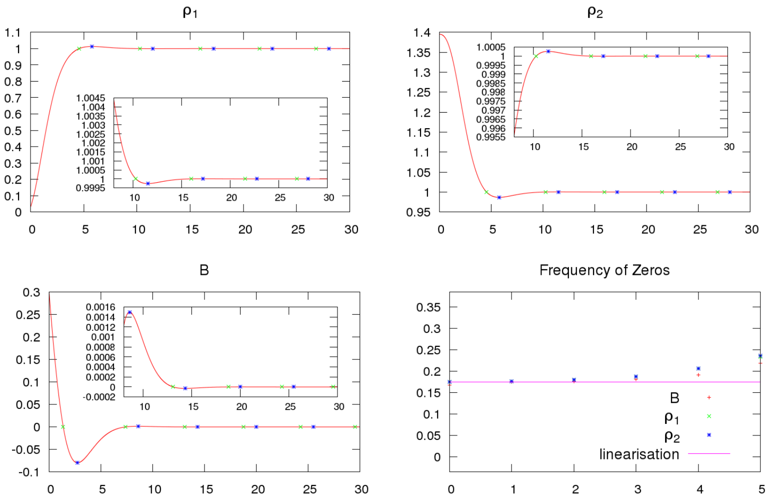

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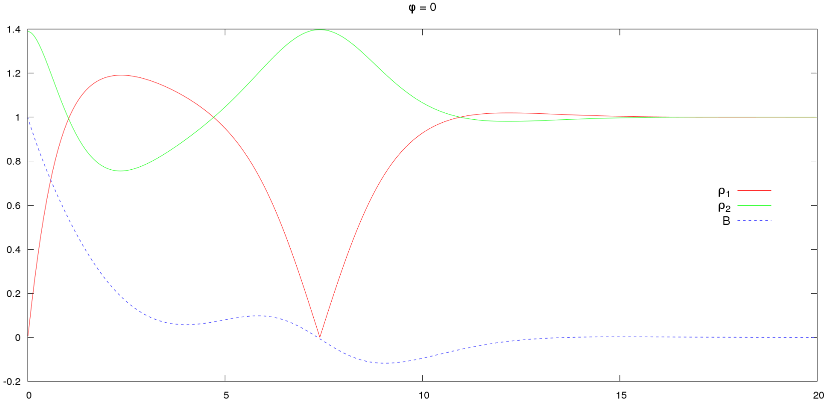

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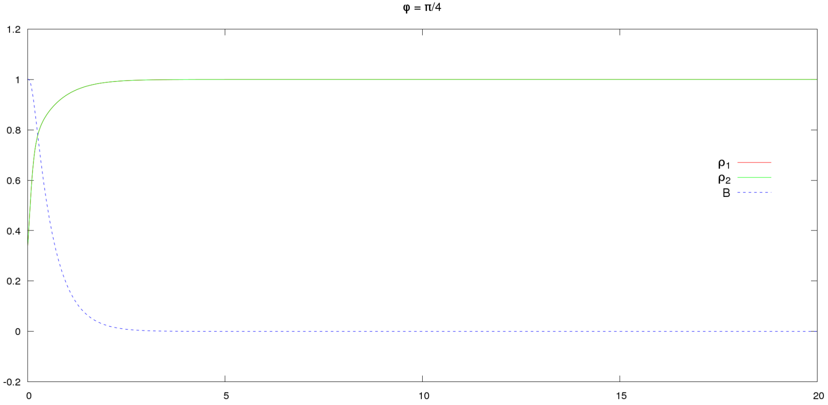

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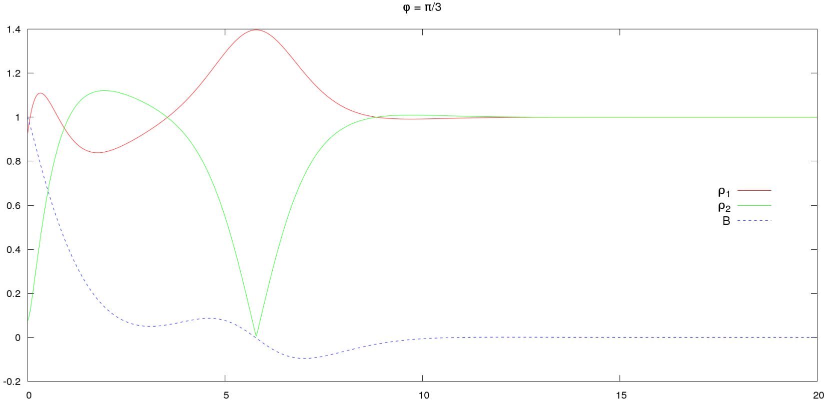

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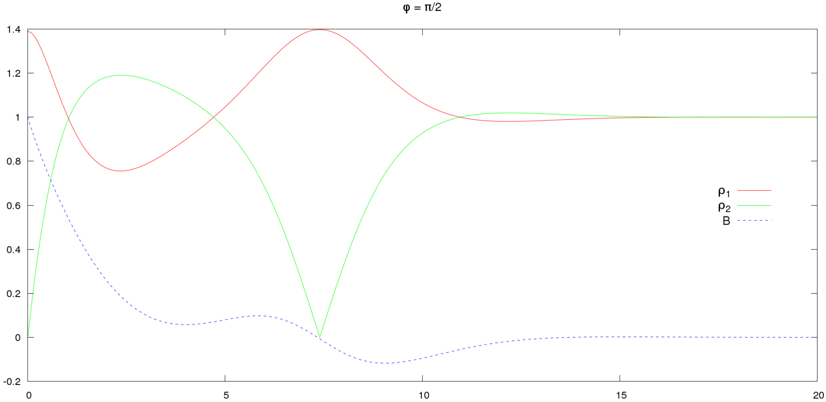
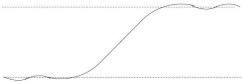


Summary

- Conventional GL theory: two independent length scales, real, orientation independent
- Anisotropy, multicomponent, BTRS:
  - all modes coupled
  - single dominant length scale
  - can be complex (spatial oscillations)
  - orientation dependent
- Applies to **any** localized defect

domain wall



vortex